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Publicly Funded Education and Human Capital in the Presence of a Convex-Concave Education Function

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Abstract: This study investigates an aggregative optimal growth model in which short-lived individuals obtain their labour skill through education. The process of human capital formation is described by an increasing, convex-concave education function relating the success rate to the educational expenditure per student. The cost of education is publicly funded by an income tax imposed on adult workers. Despite the apparent regularity and rationality of this idealized economy, it is shown that the existence of the steady state of the model is not guaranteed. In fact, the steady state only exists for carefully chosen social time rates of preference. However, the steady state, if it exists, is unique and in terms of local stability, a saddle point.

Key words: Labour skill, education, convex-concave education function, idealized economy

INTRODUCTION

The traditional theory of economic growth typically takes labour as a homogenous primary factor of production. At the same time, there has been a recognition that knowledge or skill plays an important role in augmenting raw labour productivity. In fact this is a crucial feature in the strand of the theory of endogenous growth[1-3]. Although the literature on this subject is very substantial, it seems to suffer from three main weaknesses.

Firstly and surprisingly, in spite of the obvious advantage of the overlapping generations framework in analyzing the economics of education, many studies assume that economic agents (and thus human capital) are infinitely long-lived[1,4,5]. Secondly, many models assume that either the process of acquiring knowledge is costless and instantaneous[1,6] or the time spent on education is the only opportunity cost of education[7-9]. An exception is Eicher[10] who analyzed a growth model which is free of these first two weaknesses.

Thirdly and finally, virtually all models incorporating an education sector assume that the outcome of education is certain, i.e., the possibility of failure in education is not entertained. This is essentially an implication of either the identical agents assumption or the perfect information assumption. Under the assumption of identical agents, if one student passes, then all other students of the same cohort will also pass. Under the perfect information scenario, only those students who can pass will undertake education so that the success rate is 100%. These assumptions obviously fail to take into account the facts that (i) children do not possess the same amount of natural talents and (ii) they often do not know the extent of their innate abilities until a late stage of their education.

The focus of this study is not on endogenous growth but rather on the role of education as a process of forming short-lived human capital. It follows the approach of Tran-Nam *et al*.[11] and Shimomura and Tran-Nam[12] who attempted to remedy the above drawbacks by developing a model of overlapping generations in which education takes time and incurs direct costs and its outcome is not certain. This line of approach assumes in particular that the outcome of education (in terms of the success rate) is a nonnegative, strictly increasing and strictly concave function of educational expenditure per student. The assumed concavity of the education function seems to be a reasonable one when total educational expenditure per student is large. However, it may not be entirely justified when the total educational expenditure per student is small.

It is well known that the curvature of the production function has interesting implications in models of growth. In analyzing an optimal growth model with a convex-concave production function, Skiba[13] found that there are two possible steady states: One is a saddle point while the other is an unstable focus. More recently, in the theory of endogenous growth where the production function exhibits increasing returns to scale, it has been shown that the optimal growth path exists only for carefully chosen

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values of the social discount rate[3,6,14,15]. In a model of economic growth incorporating an educational sector, there is a clear direct relationship between production and education via the produced input, skilled labour. It seems therefore appropriate to explore the role of a convex-concave education function in such models.

The principal aim of this study is to examine the implications of a convex-concave education function in an overlapping generations economy, in which education is publicly funded by an income tax imposed on workers. The crucial feature of this model is that the change in the success rate is strictly increasing (decreasing) for relatively small (large) educational expenditures per student. The main findings of this study are as follows. Despite all the regularity and rationality features of the model, the existence of a steady state of this economy is no longer assured. However, if a steady state exists, it is unique. In terms of local stability, a steady state, if it exists, is an unstable saddle point.

The Model: Consider an aggregative closed economy that commences operation in period 1 and continues over periods t = 2, 3, 4, …, extending indefinitely into the future. This economy can be described in terms of its three components: the population, the production sector and the government. The population is composed of individuals who live for exactly for three periods. These individuals study, work and retire in the three periods of their lives, respectively. The production side of the economy consists of two sectors. In the manufacturing sector, production of a single perishable final good takes place with the aid of skilled labour and unskilled labour. The final good can be consumed or invested in education. In the education sector, students combine their natural talents with the final good to form short-lived human capital. The government is a long-lived, far sighted central planner which seeks to maximize the present value of a stream of discounted utilities based on per capita consumption. In each period, the government imposes an income tax on workers to finance the free education of students. Each of three components of the economy is described in greater detail below.

At the beginning of each period t (≥1), a new generation of individuals is born. This generation, called generation t, is denoted by its date of birth and consists of N_t members. Members of the generation t live for exactly three periods. They exist during period t (when they are called the young), t+1 (when they are called the adults), t+2 (when they are called the old); and at the end of period t+2, they all die. In the initial period 1, the adults and old people exist. Thus, at any point in time, the population is composed of the young, the adults and the old; each age group associated with a different generation. It is assumed that only the adults are fertile and that the rate of fertility n is an exogenously given, nonnegative constant, i.e.:

$$N_t = (1+n)N_{t-1} \quad t = 1, 2, 3 \ldots \tag{1}$$

where, $n \geq 0$ (to avoid the uninteresting possibility of long run extinction of the population) and N_0 (≡ number of adults at time 1) is given and positive. Without serious loss, it is further assumed that the number of adults and the number of old people at time 1 are also related by Eq. (1).

All individuals are required to undertake education when young (because child labour is illegal or/and for equity reasons all children have the right to go to school). The natural talents of children are not identical but obey a fixed distribution over time. As a result, the outcome of education is not deterministic. Successful students of generations t will become skilled workers in period t+1, while their unsuccessful fellow students will work as unskilled workers in period t+1. The skill level of successful students is treated as uniform. (We need not think of the outcome of education in terms of success or failure. More broadly speaking, education can be thought of as producing two broad categories of graduates. Category 1 graduates work as skilled labour, Category 2 graduates as unskilled labour).

The success of education depends on students' natural talents and the amount of resources invested on education. Given that the distribution of natural talents remains constant over time, the overall probability that students will successfully complete their study is assumed to be dependent on the educational expenditure per student. (For simplicity, educators are assumed away.) The time-invariant functional relationship g that relates x_t (the pass rate in period t) to z_{t-1} (educational expenditure per student in period t-1) is called the education function Eq. (2):

$$x_t = g(z_{t-1}) \quad t = 2, 3, 4 \ldots \tag{2}$$

Education investment is indispensable in the sense that g(0) = 0. The education function g is supposed to be nonnegative, twice differentiable and strictly increasing and approach unity from the left as educational investment becomes indefinitely large. In particular, we also assume g to be strictly convex (concave) for $z<(>)\tilde{z}$ with $g'(\tilde{z})<\infty$.

The assumed change in curvature of the education function requires elaboration. Education is subjected to the law of diminishing returns. It typically requires some fixed costs (e.g., costs of building schools and libraries). A small increase of educational expenditure above the fixed costs is most likely to raise the success ratio at an increasing rate. As educational expenditure increases, the returns to education may decline steadily. Beyond a certain level of expenditure, any further increases will cause the success ratio to increase at a decreasing rate. Thus, it may be appropriate to assume that the rate of return to education is strictly increasing (decreasing) for relatively small (large) educational expenditure per child.

Next, let us describe the production technology prevailing in the manufacturing sector. Competitive firms in the manufacturing sector produce a perishable good with the aid of skilled and unskilled workers using an unchanging technology. The crucial difference between skilled and unskilled labourers is that skilled workers can perform unskilled tasks while unskilled workers cannot engage in skilled work. The aggregate production function is then written Eq. (3):

$$Y_t = F(N_t^s, N_t^u)t = 1, 2, 3 \ldots \tag{3}$$

where Y_t, N_t^s and N_t^u stand for the total output and the amount of skilled and unskilled labour in period t, respectively. The production function F(x, y) is assumed to possess all standard neoclassical properties, specifically, it is linearly homogeneous in skilled labour x and unskilled labour y, strictly increasing in x and y, strictly quasi-concave and satisfies the Inada conditions $\lim_{x\to 0} F_x = \lim_{y\to 0} F_y = \infty$ where F_x and F_y stand for the marginal product of skilled and unskilled workers, respectively. Skilled labour is indispensable for production, i.e., F(0, y) = 0 for all y. For completeness, the number of skilled workers in period 1 (N_1^s) is supposed to be given, positive and smaller than N_0. Full employment of labour resources requires that Eq. (4):

$$N_t^s + N_t^u = Nt - 1 \; t = 1, 2, 3 \ldots. \tag{4}$$

Keeping in mind that by definition $x_t \equiv N_t^s/N_{t-1}$ and making use the linear homogeneity of F and the full-employment condition (4), the production function can be rewritten in per worker terms as follows:

$$Y_t = N_{t-1} f(x_t) t = 1, 2, 3 \ldots \tag{5}$$

where f(x) ≡ F[x, (1-x)]. It is assumed that there exists a unique $\hat{x} \in (0, 1)$ such that Eq. (6):

$$f'(x) > (= 0) \text{ for } x < (\hat{x}) \tag{6}$$

To avoid triviality, it is further assumed that $\hat{x} > \tilde{x} \equiv g(\tilde{z})$. Condition (6) means that at $\hat{x}$ and beyond, skilled and unskilled labour are perfect substitutes.

Finally, let us describe the government's optimizing problem. The government is a long-lived, far-sighted central planner. It provides, in each period, free education to all young children and finances it expenditure by imposing an income tax on adult workers in the economy. Let τ_t be the overall tax rate in period t. The government budget is supposed to be balanced in all time periods, i.e., Eq. (7):

$$\tau_t Y_t = z_t N_t \; t = 1, 2, 3 \ldots \tag{7}$$

The government's objective is to maximize the target function Eq. (8):

$$J = \sum_{t=1}^{T} \left(\frac{1+n}{1+\rho} \right)^t U(c_t) \tag{8}$$

where, ρ (> n) is the social rate of time preference, c_t is per capita consumption in period t and U is a twice differentiable, strictly increasing and strictly concave social welfare function satisfying the regularity condition Eq. (9):

$$\lim_{c\to 0} U'(c) = \infty \tag{9}$$

Note that the distribution of consumption between the two types of labour and between the three generations in any period is also ignored. A possible interpretation is that the government can devise a tax-transfer system to guarantee that total consumption is equally divided among all members of the population at any time. Under this egalitarian approach to consumption, it does not matter whether the tax scale is proportional, regressive or progressive, or whether the process of taxing-transferring is costless or not.

Combining the balanced budget condition (7) and Eq. (5) and (1) yields Eq. (10):

$$z_t = t_t f(x_t) / (1+n) t = 1, 2, 3 \ldots \tag{10}$$

Similarly, the per capita consumption in period *t* can be expressed as Eq. (11):

$$c_t = (1 - t_t) / f(x_t) / h(n) t = 1, 2, 3 \ldots \tag{11}$$

where, h(n) ≡ 2+n+1/(1+n).

Thus, the government's optimizing problem can be cast in the form of a discrete-time finite-horizon control problem as follows eq. (12):

$$\max J \equiv \sum_{t=1}^{T} (1+r)^{-t} U(c_t) \tag{12}$$

Subject to Eq. (13-15):

$$x_t = g[\tau_t - 1 f(x_t - 1)/(1+n)] t = 2,3,\ldots,T \tag{13}$$

$$x_1 \text{ is given and } x_{T+1} \text{ is free} \tag{14}$$

$$0 \le \tau_t \le 1 \text{ and } 0 \le x_t \le 1_t = 2,3,\ldots,T \tag{15}$$

where, $r \equiv (\rho\text{-}n)/(1+n)$ (>0), τ_t is the control variable and x_t the state variable.

Dynamic analysis: We note that $J \equiv J(\tau_1, x_1, \tau_2, x_2, \ldots, \tau_{T-1}, x_{T-1}, \tau_T, x_T)$ is a continuous function defined on a compact domain $[0, 1]^{2T}$ (from Tychonoff's theorem we know that a finite product of a compact set is itself a compact set). Keeping in mind the strict concavity of U and the monotonicity of g, the Weierstrass theorem ensures that there exists a unique solution $\{(\tau_t^*, x_t^*)$: t = 1, 2, …, T$\}$ to problem (12)-(15). Further, this optimal path must be an interior solution in the sense that $0<\tau_t^*<1$ and $0<x_t^*<1$ for t = 1, 2, …, T (since if $\tau_t = 0$ then $c_{t+1} = 0$ and if $\tau_t = 1$ then $c_t = 0$ and neither of which can be optimal in view of the regularity condition (9)). Thus, we can now state.

Proposition 1: An interior solution to problem (12)-(15) exists and is unique.

To characterize the optimal time path we first define the augmented objective function as Eq. (16):

$$\begin{aligned} J_a &\equiv \sum_{t=1}^{T} \left\{ (1+r)^{-t} U(c_t) - p_{t+1}\left[x_{t+1} - g(z_t) \right] \right\} \\ &\equiv \sum_{t=1}^{T} \left(H_t - p_{t+1} x_{t+1} \right) \end{aligned} \tag{16}$$

where, p_t is the co-state variable and H_t ($\equiv (1+r)^{-t}U(c_t) + p_{t+1}g[\tau_t f(x_t)/(1+n)]$) is the discrete Hamiltonian. The first-order necessary conditions for an interior optimum are Eq. (17-19):

$$\begin{aligned} p_t &= \partial H_t / \partial x_t = (1+r)^{-t} (1-\tau_t) f' / h(n) \\ &+ p_{t+1} g' \tau_t f' / (1+n) t = 1,2,\ldots,T \end{aligned} \tag{17}$$

$$\begin{aligned} &\partial H_t / \partial \tau_t = (1+r)^{-t} f / h(n) \\ &+ p_{t+1} g' f / (1+n) = 0 \ t = 1,2,\ldots,T \end{aligned} \tag{18}$$

$$p_{T+1} = 0 \tag{19}$$

where, the *'s are omitted to alleviate cumbersome notation.

Equations (16) and (17) supply 2T-1 optimality conditions while (19) gives one tranversality condition. These equations, together with T-1 feasibility conditions in (13) and one boundary condition (14) completely determine the values of the 3T variables $(\tau_t^*, x_t^*, p_{t+1})$, t = 1, 2, …, T. Since the optimal path is unique, these 3T equations provide necessary and sufficient conditions for a unique global optimum. Once τ_t^* and x_t^* are determined, the value of c_t^* is specified by (11).

Solving for p_{t+1} from (18) and substituting it into (17) gives after some simplification Eq. (20):

$$\begin{aligned} &U'(c_t^*)/U'(c_{t+1}^*) = (1+\rho)/[g'(z_{t-1}^*) f'(x_t^*)] \\ &t = 2,3,\ldots,T+1 \end{aligned} \tag{20}$$

The economic interpretation of (20) is abundantly clear. By reducing per capita consumption by one unit of the good in period t-1, the loss in social welfare is U'(ct-1). Investing this one unit of good in education raises xt by g'(zt-1)h(n)/(1+n), which in turns gives rise to an increase of f'(xt)g'(zt-1)h(n)/(1+n) in per worker output in period t. The corresponding increase in social welfare in period t is therefore given by $U'(c_t)f'(x_t)g'(z_{t-1})/(1+\rho)$. Equation (20) is thus the familiar competitive intertemporal arbitrage condition that equates the marginal utility of consumption in two periods and the marginal rate of substitution of consumption between the two periods.

From equation (20) it is also clear that f' (x_t^*) > 0 or $x_t^* < \hat{x}$ for all t > 1. Thus, we may state

Proposition 2: Except possibly at t = 1, the values of x_t^* along the optimal time path are always smaller than $\hat{x}$.

This is reminiscent of the well-known proposition in the traditional growth theory that the optimal capital/labour ratio k* is less than k_g where k_g is the capital/labour which maximizes per capita consumption. However, Proposition 2 allows a more straightforward interpretation. By definition, f' (x) ≡ F_x-F_y, i.e., f' (x) is the difference between marginal products of skilled and unskilled labour. If $x_t \geq \hat{x}$ then f' (x) = 0, i.e., $F_x = F_y$. This means that education is wasteful and therefore suboptimal.

Multiplying both sides of (20) by $U'\left(c^*_{t-1}\right)$ and inverting, we can express c^*_t in terms of τ^*_{t-1} and x^*_{t-1} as follows Eq. (21):

$$c^*_t = V\left\{\frac{(1+\rho)U'\left[\frac{(1-\tau^*_{t-1})f(x^*_{t-1})}{h(n)}\right]}{g'\left[\frac{\tau^*_{t-1}f(x^*_{t-1})}{1+n}\right]f'\left[g\left(\frac{\tau^*_{t-1}f(x^*_{t-1})}{1+n}\right)\right]}\right\} t = 2, 3, \ldots, T \quad (21)$$

where $V \equiv (U')^{-1}$ and V(y) > 0 and V' (y) < 0 for all y > 0. Bearing (11) in mind, (21) implies Eq. (22):

$$\tau^*_t = 1 - \frac{h(n)}{f\left[g\left(\frac{\tau^*_{t-1}f(x^*_{t-1})}{1+n}\right)\right]}V\left\{\frac{(1+\rho)U'\left[\frac{(1-\tau^*_{t-1})f(x^*_{t-1})}{h(n)}\right]}{g'\left[\frac{\tau^*_{t-1}f(x^*_{t-1})}{1+n}\right]f'\left[g\left(\frac{\tau^*_{t-1}f(x^*_{t-1})}{1+n}\right)\right]}\right\} t = 2, 3, \ldots, T \quad (22)$$

We note that (22) and (13) define the optimal time path by a system of two nonlinear difference equations. Thus we have established.

Proposition 3: The optimal time path is defined by:

$$\tau^*_t = G\left(\tau^*_{t-1}, x^*_{t-1}\right) t = 2, 3, \ldots, T \quad (22')$$

$$x^*_t = H\left(\tau^*_{t-1}, x^*_{t-1}\right) t = 2, 3, \ldots, T \quad (13')$$

where, *G* and *H* are defined in (22) and (13) respectively.

Steady state analysis: When *T* becomes indefinitely large, the steady state of the model, if it exists, is a solution to the timeless version of (13) and (20):

$$x = g[\tau f(x)/(1+n)] \quad (23)$$

$$g'[\tau f(x)/(1+n)]f'(x) = 1+\rho \quad (24)$$

Since *g* is monotonic, ϕ ($\equiv g^{-1}$) exists. Inverting both sides of (23) gives:

$$\tau f(x)/(1+n) = \phi(x) \quad (25)$$

Substituting (25) into the left hand side of (24) reduces the system (23)-(24) to:

$$\Phi(x) = 1+\rho \quad (26)$$

where, $\Phi(x) \equiv g'[\phi(x)]f'(x)$. Since $g'[\phi(x)] \equiv 1/\phi'(x)$, equation (26) can be equivalently written as:

$$\phi'(x) = f'(x)/(1+\rho) \quad (26')$$

Once x^* is determined by (26), τ^* can be worked out by (25). The timeless version of (11) can then be used to pin down the value of c^*. It is interesting to note that the determination of the steady state involves the social time rate of preference but is completely independent of the functional form of the social utility *u*.

Interpretation of the steady state: Let us assume for the time being that there exists a unique x^* that satisfies (26). Since $\phi(x)$ can be thought of the total cost of education, $\phi'(x)$ is simply the marginal cost of education. As discussed previously, f' (*x*) represents the marginal benefit of education. Equation (26') thus gives the familiar first-order condition of optimality equating the marginal cost of education (in the current period) to the marginal benefit of education (in the next period). Under the assumption about the curvature of *g*, $\phi'(x)$ first declines, attains a minimum at $\tilde{x}$ and then rises. The second-order condition requires that the $\phi'(x)$ curve cuts the $f'(x)/(1+\rho)$ curve on the rising portion of $\phi'(x)$, which in turn implies that the steady state lies on the concave range of the education function ($x^* > \tilde{x}$). It is now possible to state.

Proposition 4: At a steady state, if one exists, $x^* > \tilde{x}$ and the marginal cost of education is equal to the (discounted) marginal benefit of education.

Existence and uniqueness of the steady state: To determine the existence of a steady state we need to study the conditions under which the curve $\phi'(x)$ intersects the curve $f'(x)/(1+\rho)$ on the rising portion of the $\phi'(x)$. Now, under the assumptions made, the $f'(x)/(1+\rho)$ curves is asymptotic to the vertical axis, declines steadily and becomes horizontal at $x = \hat{x}$. The curve $\phi'(x)$ has the U shape, attaining a minimum at $\tilde{x}$ (recalling that $\tilde{x} < \hat{x}$). Note that $\phi'(x)$ may approach infinity or a finite constant as *x* approaches 0^+, depending respectively on whether $g'(z)$ approaches 0 or a finite constant *z* approaches 0^+.

There are in general three cases:

Case i: $\phi'(x)$ approaches a positive constant, or infinity at a slower rate than $f'(x)$, i.e., $\Phi(x)$ approaches infinity as x approaches 0^+; or

Case ii: $\phi'(x)$ and $f'(x)$ approach infinity at the same rate, i.e., $\Phi(x)$ approaches a positive constant as $x \to 0^+$; or

Case iii: $\phi'(x)$ approaches infinity at a faster rate than $f'(x)$ so that $\Phi(x)$ approaches zero as $x \to 0^+$.

Only in case i the intersection of the $\phi'(x)$ and $f'(x)/(1+\rho)$ curves is guaranteed for any choice of ρ. In cases ii and iii, such a intersection occurs if and only if $1+\rho \le \max \Phi(x)$.

Note further that in case i the $\phi'(x)$ and $f'(x)/(1+\rho)$ curves only intersects once. However, this intersection corresponds to a steady state only if the resulting x^* is greater than $\tilde{x}$, as dictated by the second-order condition. In cases ii and iii, if $1+\rho = \max \Phi(x)$, then the $\phi'(x)$ curve will touch the $f'(x)/(1+\rho)$ curve once. However, this happens in the falling range of the $\phi'(x)$ curve and cannot correspond to a steady state. In cases ii and iii, if $1+\rho < \max \Phi(x)$, the two curves will intersect twice. However, this gives rise to at most one steady state at which $x^* > \tilde{x}$. We are now able to state

Proposition 5: A steady state exists if and only if $1+\rho \le \max \Phi(x)$ and the intersection of $\phi'(x)$ and $f'(x)/(1+\rho)$ curves occur in the concave range of the education function. If a steady state exists, it is unique.

The interpretation of the above proposition is not straightforward. In the Uzawa-Lucas-Romer strand of the theory of endogenous growth where production exhibits increasing returns it has been established that various equilibria only exist for carefully chosen time rates of preference[3,6,14,15]. Proposition 5 is somewhat analogous to these results. In the present study, the magnitude of $\Phi(x)$ is indicative of the ratio of society's future gain to its sacrifice of current consumption for education. If the social discount factor exceeds the highest possible intertemporal tradeoff ratio, the optimal path will never converge to a steady state, in spite of all the regularity and rationality features of the model.

Comparative statics of the steady state: Since a steady state, if one exists, is unique, it is meaningful to talk about its comparative statics, so long as the change is small so that the solution to (26) remains on the rising portion of $\phi'(x)$. Now, differentiating (25) with respect to x^* and making use of (26') we have:

$$\frac{d\tau^*}{dx^*} = [(1+n)(1+\rho) - \tau^*]\frac{f'(x^*)}{f(x^*)} > 0 \tag{27}$$

That is, τ^* and x^* move in the same direction. In view of how the $\phi'(x)$ and $f'(x)/(1+\rho)$ curves respond to changes in the model parameters, we can therefore establish.

Proposition 6: The greater the rate of population growth (higher n), the lower is the steady state tax rate τ^* and the lower is the steady state ratio of skilled labour x^*. The greater the social rate of discount (higher ρ), the lower is the steady state tax rate τ^* and the lower is the steady state ratio of skilled labour x^*. An improvement in the education technology will result in higher τ^* and x^*. An improvement in favour of skilled labour in the manufacturing technology will result in higher τ^* and x^*.

The above results are intuitively appealing and need no further elaboration.

Local stability of the steady state: Suppose now that a steady state (τ^*, x^*) exists uniquely. To study the local stability of the solution path, we rewrite (22') and (13') respectively as:

$$K\left(\tau_t, x_t; \tau_{t-1}, x_{t-1}\right) = 0 \tag{22''}$$

$$L\left(\tau_t, x_t; \tau_{t-1}, x_{t-1}\right) = 0 \tag{13''}$$

where, $K \equiv U'(c_t)g'(z_{t-1})f'(x_t) - (1+\rho)U'(c_{t-1})$ and $L \equiv x_t - g[\tau_{t-1}f(x_{t-1})/(1+n)]$. Linearizing (22'') and (13'') about (τ^*, x^*) we have in matrix notation:

$$\begin{bmatrix} x_t - x^* \\ \tau_t - \tau^* \end{bmatrix} = \begin{bmatrix} -L_3 & -L_4 \\ (K_1L_3 - K_3)/K_2 & (K_1L_4 - K_4)/K_2 \end{bmatrix} \begin{bmatrix} x_{t-1} - x^* \\ \tau_{t-1} - \tau^* \end{bmatrix} \tag{28}$$

where, $L_3 = -\tau g' f' / (1+n)$, $L_4 = -g' f / (1+n)$, $K_1 = (1-\tau)U''g'(f')^2/h + U'g'f''$, $K_2 = -(1+\rho)U''f/h$, $K_3 = \tau U'g''(f')^2/(1+n) - (1+\rho)(1-\tau)U''f'/h$ and $K_4 = U'g''ff'/(1+n) + (1+\rho)U''f/h$, all evaluated at ($\tau^*$, x^*).

It can then be shown that the characteristic equation arising from the above system is given by:

$$\lambda^2 - (2 + r + \beta) + (1 + r) = 0 \tag{29}$$

where, $\beta \equiv h\, U'g''/[(1+n)U''\, g']$. It can be shown that the two characteristics roots are:

$$\lambda_1 = 1 + r + hU'g''/[(1+n)U''g'] \qquad (30)$$

And

$$\lambda_2 = 1 - \left(\sqrt{(r+\beta)^2 + 4\beta} - (r+\beta)\right)/2 \qquad (31)$$

where, $0 < \sqrt{(r+\beta)^2 + 4\beta} - (r+\beta) < 2$. Keeping in mind that $x^* > \tilde{x}$ so that $g'' < 0$, we have $0 < \lambda_2 < 1 < \lambda_1$. We may now state

Proposition 7: The steady state, if it exists, is an unstable saddle point.

It is interesting to note that if the $\phi'(x)$ and $f'(x)/(1+\rho)$ curves intersect twice, then the intersection in the convex range of the education function (thus not a steady state) can be an unstable focus, node or saddle point (see the Appendix).

CONCLUSION

This study constructs a model of an idealized economy in which the population is composed of individuals who live for exactly three periods. When young, all individuals are required to undertake publicly provided education. As adults, they work as skilled or unskilled labour, depending on the outcome of their education, to produce a single perishable good which can be used for consumption or education. When old, they are all retired. The economy is governed by a long-lived central planner which conducts a tax-transfer scheme to finance the costs of public education and to redistribute income so that the distribution of consumption is egalitarian. The government's long-term objective is to maximize the present value of a sum of (discounted) social utilities based on per capita consumption.

A special feature of the model is the education sector. Education as a process of human capital formation is not only costly, in terms of time spent by students and resources expended, but its outcome is uncertain, depending in part on the distribution of talents. In this sense, the model rejects the assumption of identical agents. The education process produces two broad kinds of input for production: skilled and unskilled labour. In particular, assuming a fixed distribution of natural abilities, the ratio of skilled labour to the workforce in the current period strictly increases in educational expenditure per student in the last period. The law of diminishing returns applies to education so that as educational expenditure per student increases, the success rate first rises at an increasing rate and then at a decreasing rate.

Apart from the education sector, the rest of the economy is characterized by the usual regularity and rationality assumptions: the aggregate production function possesses all the standard neoclassical properties; specifically, the central government is far sighted and there are no transaction costs. Under these assumptions, it is shown that an interior, optimal time path exists uniquely. Surprisingly, however, the optimal time path will converge to a unique and unstable saddle-point steady state under fairly restricted conditions. Further, the steady state, if it exists, is independent of the functional form of the social utility (but not the social time rate of discount). These results are counter-intuitive within the neoclassical framework of analysis.

Appendix: The characteristic equation is given by:

$$\lambda^2 - (2+r+\beta)\lambda + (1+r) = 0$$

where, $\beta \equiv h(n)U'g''/[(1+n)U''g']$. The discriminant of (12) is $\Delta = (r+\beta)^2 + 4\beta$ and the product of the characteristic roots satisfies:

$$\lambda_1\lambda_2 = 1 + r > 1$$

Suppose that the $\phi'(x)$ intersects the curve $f'(x)/(1+\rho)$ twice, at x_1^* and x_2^* where $x_1^* < \tilde{x} < x_2^*$. At the steady state x_2^* it has been shown that $0 < \lambda_2 < 1 < \lambda_1$. At x_1^*, $\beta < 0$ and the discriminant Δ can be negative, zero or positive.

- $\Delta < 0$

In this case there are two complex roots $\{2+r+\beta \pm i[-(r+\beta)^2-4\beta]^{1/2}\}/2$ with $|\lambda_1| = |\lambda_2| = \sqrt{1+r} > 1$, i.e., (τ_1^*, x_1^*) is an unstable focus (a spiral point).

- $\Delta = 0$

In this case there is a repeated real roots $\lambda = (2+r+\beta)/2$ with $|\lambda_1| = \sqrt{1+r} > 1$, i.e., (τ_1^*, x_1^*) is an unstable improper node.

- $\Delta > 0$

There are two subcases:

$$r + \beta > 0$$

Since $\Delta < (r+\beta)^2$ we may write $\sqrt{\Delta} = r+\beta-\theta$ where $0 < \theta < r+\beta$. There are two real characteristics roots:

$$\lambda_1 \frac{2 + r + \beta + (r + \beta - \theta)}{2} = 1 + r\beta - \frac{\theta}{2} > 1$$

And:

$$\lambda_2 \frac{2 + r + \beta - (r + \beta - \theta)}{2} = 1 + \frac{\theta}{2} > 1$$

The point (τ_1^*, x_1^*) is in this case an unstable node.

$r+\beta < 0$

Suppose $r+\beta = -k$ ($k > 0$). The requirement $\Delta > 0$ implies that $-4\beta < (r+\beta)^2 = k^2$, i.e., $\beta > -k^2/4$. Since r is positive this in turn implies that $-k = r + \beta > \beta > -k^2/4$, i.e., $k > 4$. In other words, $-4 \leq r+\beta < 0$ is incompatible with $\Delta > 0$. Thus, we only need to consider $r+\beta < -4$. Since $\lambda_1\lambda_2 = 1 + r > 1$ and $\lambda_1+\lambda_2 = 2 + r + \beta < -2$, both characteristic roots are negative. In this case, we have either $|\lambda_1| < 1 < |\lambda_2|$ (a saddle point) or $1 < |\lambda_1| < |\lambda_2|$ (an unstable focus). In his study, Skiba[13] showed that if the steady state occurs in the convex range of the production function, then it is an unstable focus. The results above are similar to but richer than Skiba's findings.

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