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Chaotic Attitude Tumbling of Satellite in Magnetic Field

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Abstract: In this study, the half width of the chaotic separatrix has been estimated by chrikov's criterion. Through surface of section method, it has been observed that the magnetic torque parameter, the eccentricity of the orbit and the mass distribution parameter play an important in changing the regular motion into chaotic one.

Key words: Chaos, celestial mechanics, solar system, periodic orbits, poincare section

INRODUCTION

Bhardwaj[1] has discussed chaos in non-linear planar oscillation of a satellite under the influence of third-body torque. Sidlichovský[2] has discussed the existence of a chaotic region which is formed by trajectories crossing a critical curve which corresponds to the separatrix of fast pendulum motion. Tiscareno[3] carried out extensive numerical orbit integrations to probe the long-term chaotic dynamics of the 2:3 (Plutinos) and 1:2 (Twotinos) mean motion resonances with Neptune. Kauprianov and Shevchenko[4] studied the problem of observability of chaotic regimes in the rotation of planetary satellites. Contopoulos and Efstathiou[5] studied Escapes and Recurrence in a Simple Hamiltonian System. They studied a simple dynamical system with escapes using a suitably selected surface of section. Selaru *et al*.[6] studied Chaos in Hill's generalized problem from the solar system to black holes. Carruba *et al.*[7] have discussed Chaos and Effects of Planetary Migration for the Saturnian Satellite Kiviuq.

Equation of motion: The equation of motion for the non-linear motion of a satellite under the influence of magnetic torque in an elliptic orbit as obtained as

$$\frac{d^2\theta}{dt^2}+\frac{\mu}{2r^3}n^2\sin\delta-\frac{\varepsilon\mu}{2r^3}\sin 2(\Omega-\alpha_m+v)=0 \qquad (1)$$

which can also be written as

$$(1+e\cos v)\frac{d^2q}{dv^2}-2e\sin v\frac{dq}{dv} -4e\sin v+n^2\sin q=\varepsilon\sin(a_1+bv) \qquad [1]$$

Estimation of resonance width: As r and v are periodic in time and as $\theta=v+\frac{\delta}{2}$, using Fourier like Poisson-Series as discussed in Bhardwaj and Tuli[1], Equation (1), becomes

$$\frac{d^2\theta}{dt^2}+\frac{{w_0}^2}{2}\sum H\left(\frac{m}{2},e\right)\sin(2\theta-mt)-\frac{\varepsilon}{2}\sum H\left(\frac{m}{b},e\right)\sin\left(2(\Omega_0-\alpha_1)+\frac{mt}{b_1}\right)=0 \qquad (2)$$

The half integers $\frac{m}{2}$ will be denoted by p. Resonances occur whenever one of the arguments of the sine or cosine functions is nearly stationary i.e., whenever $\left|\frac{d\theta}{dt}-p\right|<<\frac{1}{2}$. Using slowly varying resonance variable, $v_p=\theta-pt$, holding v_p fixed and averaging for small w_0, then, Equation (2) becomes

$$\frac{d^2v_p}{dt^2}+\frac{{w_0}^2}{2}H(p,e)\sin 2v_p-\frac{\varepsilon}{2}H\left(\frac{p}{b},e\right)\sin 2\left(\Omega_0-\alpha_1+\frac{2pt}{b}\right)=0 \qquad (3)$$

which is a equation of perturbed pendulum perturbed by a force $\frac{\varepsilon}{2}H\left(\frac{p}{b},e\right)\sin 2\left(\Omega_0-\alpha_1+\frac{2pt}{b}\right)$

When $\varepsilon\neq 0$ the Equation.(3) becomes

$$\frac{d^2x_p}{dt^2}+f'(x_p)=m_p\phi'(t,c) \qquad (4)$$

Where

$$f'(x_p)={k_{1p}}^2\sin x_p,\ m_p={k_{2p}}^2,\ \phi'(t,c)=\sin\frac{4p}{b}(t+a_3)$$

$$x_p=2v_p,$$

$${k_{1p}}^2={w_0}^2H(p,e),\ {k_{2p}}^2=\varepsilon H\left(\frac{p}{b},e\right),\ a_3=\frac{(\Omega_0-\alpha_1)b}{2p}$$

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For the unperturbed part of Equation (4), $\left(\frac{dx_p}{dt}\right)^2 = c_{1p} + 2k_{1p}^2 \cos x_p$, where c_{1p} is constant of integration. The motion to be real if $c_{1p} + 2k_{1p}^2 \geq 0$. There are three Categories of motion depending upon $c_{1p} > 2k_{1p}^2, c_{1p} < 2k_{1p}^2$ and $c_{1p} = 2k_{1p}^2$

Category-I $c_{1p} > 2k_{1p}^2$

If $\frac{dx_p}{dt} \neq 0$, the motion is said to be revolution and unperturbed solution is

$$x_p = l_p + c_{1p} \sin l_p + O(c_{1p}^2),$$

where, $l_p = n_p t + \varepsilon_1,\ c_{1p} = \frac{k_{1p}^2}{n_p^2}$ and

$$\frac{1}{n_p} = \frac{1}{2\pi}\int_0^{2\pi} \frac{dx_p}{(c_{1p} + 2k_{1p}^2 \cos x_p)^{1/2}},$$

c_{1p}, ε_1 are arbitrary constants and l_p is an argument. Using theory of variation of parameters, since m_p *and* k_{1p}^2 are small quantities, so rejecting second or higher order terms $\frac{dc_{1p}}{dt} \cong 0 \Rightarrow c_{1p}$ is a constant upto second order of approximation.

and $\frac{d^2 l_p}{dt^2} \cong m_p \sin\frac{4p}{b}(t + a_3) \cong 2m_p \sin\frac{2pl_p}{bn_p}$. If we take $\frac{2pl_p}{bn_{p0}} = x_p$, then

$\left(\frac{dx_p}{dt}\right)^2 = c_{2p} + 2k_{3p}^2 \cos x_p$, where c_{2p} is constant of integration and $k_{3p}^2 = \frac{4\varepsilon p}{bn_{p0}}$.

Again, we get three types of motion, Type I, II is that in which $\frac{dx_p}{dt}$ >0, <0, Type III is that in which $\frac{dx_p}{dt}$ =0, at 0 or π,

For type-I, our solution is

$$x_p = N_p t + \varepsilon_2 + \frac{k_{3p}^2}{N_p^2}\sin(N_p t + \varepsilon_2) + \frac{k_{3p}^4}{8N_p^4}\sin 2(N_p t + \varepsilon_2) + \ldots\ldots\ldots$$

where $\frac{1}{N_p} = \frac{1}{2\pi}\int_0^{2\pi} \frac{dx_p}{(c_{2p} + 2k_{3p}^2 \cos x_p)^{1/2}}$, which is the case of revolution.

For the type II, the solution is $x_p = \lambda \sin(p't + \lambda_0)$ where $p' = 2\sqrt{\frac{\varepsilon p}{bn_{p0}}}$.

λ *and* λ_0 being arbitrary constants. This is the case of liberation.

Type III occurs when $c_{2p} = 2k_{3p}^2 = \frac{8\varepsilon p}{bn_0}$

The solution is $x_p + \pi = 4\tan^{-1} e^{k_{3p}t} + \alpha_0$, where α_0 is an arbitrary constant and the other having a particular value. When $t \to \pm\infty,\ x_p \to \pm\pi$, at both places, $\left(\frac{dx_p}{dt}\right) = 0$ and all higher derivatives of x_p approach to zero. This is the case of infinite period separatrix which is asymptotic forward and backward in time to the unstable equilibrium.

Category II: $c_{1p} < 2k_{1p}^2$

In this case unperturbed solution is

$$x_p = c_{1p} \sin l_p + \frac{c_{1p}^3}{192}\sin 3l_p + \ldots\ldots\ldots\ldots$$

where,

$l_p = n_p t + \varepsilon_1$,

$n_p = k_{1p}\left[1 - \frac{1}{16}c_{1p}^2 + \ldots\ldots\ldots\right]$, c_{1p} *and* ε_1 are arbitrary constants.

In case of perturbed equation, we get

$k = n_p c_{1p} \Rightarrow k \cong k_{1p} c_{1p}$ and

$$\frac{dc_{1p}}{dt} \cong \frac{m_p}{k_{1p}}\cos l_p \sin\frac{4p}{b}(t + a_3)$$

Now, $\frac{dl_p}{dt} \cong k_{1p} - \frac{m_p}{k_{1p}c_{1p}}\sin l_p \sin\frac{4p}{b}(t + a_3)$

$$\frac{d^2 l_p}{dt^2} \cong -\frac{4pm_p}{bk_{1p}c_{1p}}\sin l_p \cos\frac{4p}{b}\left(\frac{l_p - \varepsilon_1}{n_p} + a_3\right)$$

In the first approximation of $n_p = n_{p0},\ c_{1p} = c_{1p0}$, we get

$$\frac{d^2 l_p}{dt^2} \cong -\frac{4pm_p}{bk_{1p}c_{1p0}}\sin l_p \cos\frac{4p}{b}\left(\frac{l_p - \varepsilon_1}{n_{p0}} + a_3\right)$$

As a special case, let us assume that

$$\frac{2p}{b}\left(\frac{l_p - \varepsilon_1}{n_{p0}} + a_3\right) = \frac{n_1\pi}{2}, n_1 \in I.$$

When n_1 is odd, then,

$$\frac{d^2 l_p}{dt^2} + k_{4p}^2 \sin l_p = 0,\quad k_{4p}^2 = -\frac{4pm_p}{bk_{1p}c_{1p0}} > 0\ as\, m < 0$$

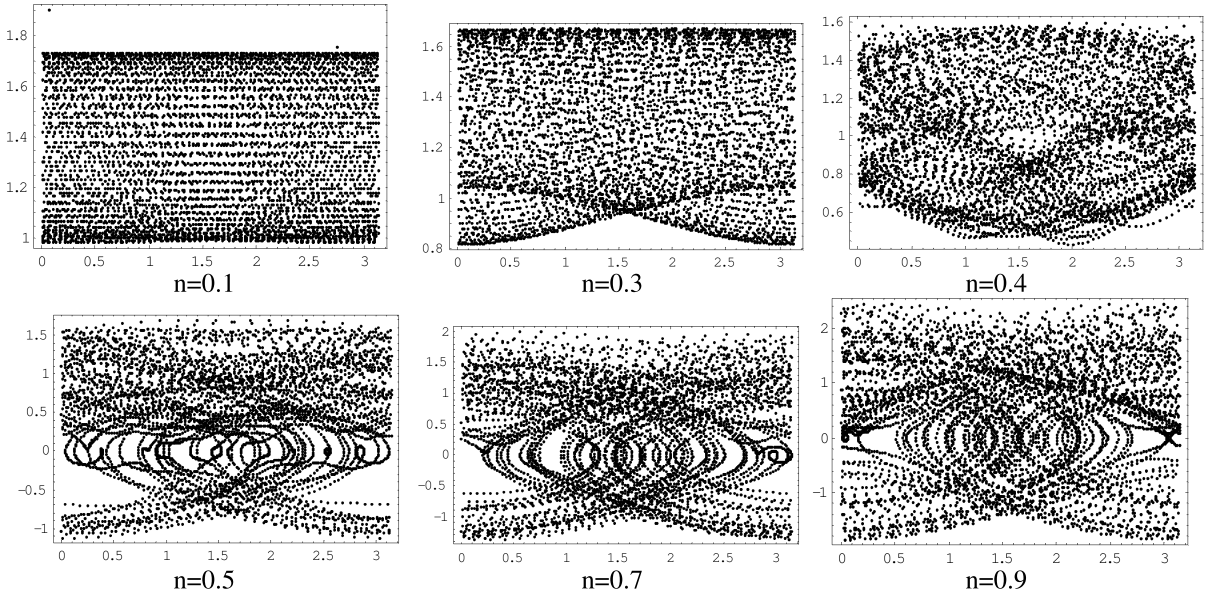

Fig. 1: Surface of sections for e = 0.0549, $\varepsilon = 0.001$, a_1=0.1153 at n=0.1, 0.3, 0.4, 0.5, 0.7, 0.9

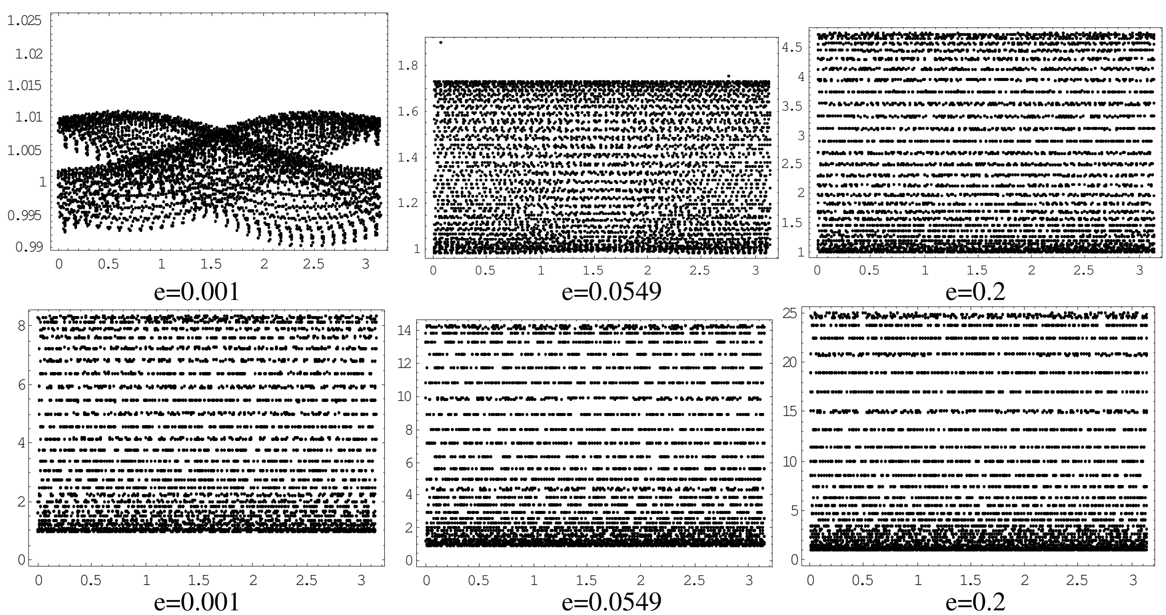

Fig. 2: Surface of sections for n=0.1, $\varepsilon = 0.001$, a_1=0.1153 at e=0.001, 0.0549, 0.2, 0.3, 0.4, 0.5

which is again the equation of pendulum. As in previous case this equation gives us revolution, liberation and infinite period separatrix motion.

On the other hand, if n_1 is even, then,

$$\frac{d^2 l_p}{dt^2} = -\frac{4pm_p}{bk_{1p}c_{1p0}} \sin l_p = k_{4p}{}^2 \sin l_p$$

When l_p is small, the solution of above equation is given by $l_p = e^{k_{4p}t} + e^{-k_{4p}t}$.

Category III: $c_{1p} = 2k_{1p}{}^2$

The unperturbed solution is $x_p + \pi = 4\tan^{-1} e^{k_{1p}t} + \alpha_0$, where α_0 is an arbitrary constant and the other having a specific value. This is the case of infinite period separatrix which is asymptotic forward and backward in time to the unstable equilibrium.

Near the infinite period separatrix broadened by the high frequency term into narrow chaotic band[9], for small n, the half width of the chaotic separatrix is given by $\omega_1 = \frac{I_1 - I_1^s}{I_1^s} = 4\pi\varepsilon \frac{1}{n^3} e^{-(\pi/2n)}$. Here, ω_1 increases both with ε and *n*. An estimate of *n* at which the wide

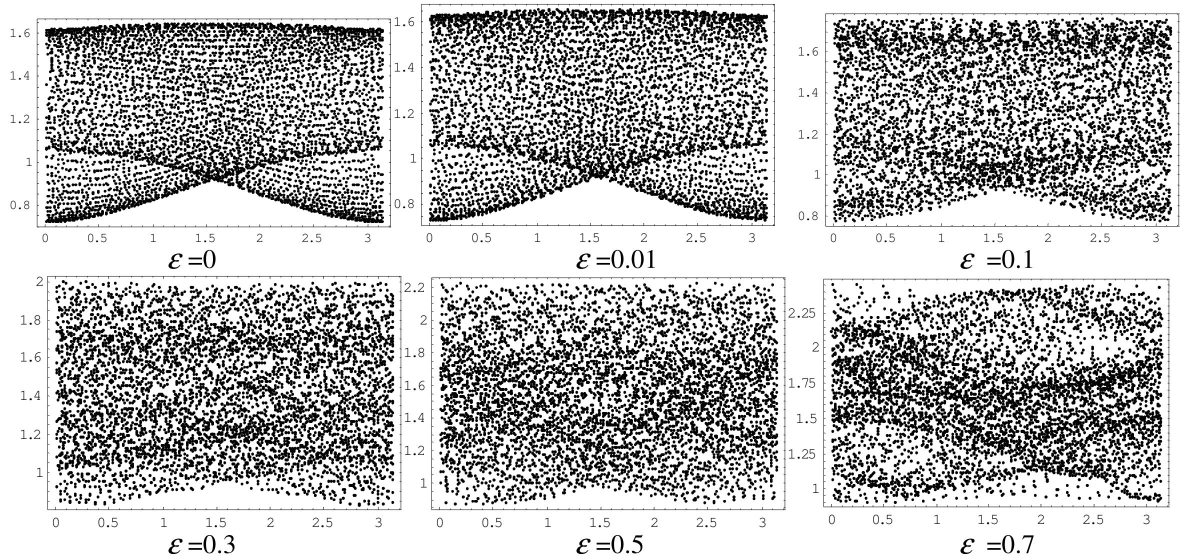

Fig. 3: Surface of sections for n=0.347, e=0.0549, a_1=0.1153 at ε=0, 0.01, 0.1, 0.3, 0.5, 0.7

spread chaotic behaviour can be observed is given by using the Chrikov's overlap criterion. This criterion states that when the sum of two unperturbed half-widths equals the separation of resonance centers, large-scale chaos ensues. In the spin-orbit problem the two resonances with the largest widths are the p = 1 and p = 3/2 states. For these two states the resonance overlap criterion becomes

$$n^{RO}\sqrt{|H(1,\, e)|} + n^{RO}\sqrt{|H(3/2, e)|} = \frac{1}{2} \quad or$$

$$n^{RO} = \frac{1}{2+\sqrt{14e}}.$$

For e = 0.0549 the mean eccentricity of Artificial satellite, the critical value of n above which large-scale chaotic behaviour is expected is n^{RO} = 0.347.

The spin orbit phase space: Using Poincare surface of section by looking at the trajectories stroboscopically with period 2π. The section has been drawn with versus v at every periapse passage. Since the orientation denoted by θ is equivalent to the orientation denoted by $\pi + \theta$, we have, therefore, restricted the interval from 0 to π. In Fig. 1-3, we have plotted $\frac{d\theta}{dv}$ versus θ, at every periapse passage. It may be observed that the chaotic separatrix surrounds each of the resonance states and each of these chaotic zones is separated from others by non-resonant quasi-periodic rotation trajectories. From Fig. 1-3, it is observed that as n, e, ε increases, the regular curves disintegrate respectively and this disintegration increases with the increase in n, e, ε.

CONCLUSION

It is also observed that the magnetic torque plays a very significant role in changing the motion of revolution into liberation or infinite period separatrix. The half width of the chaotic sepratrices estimated by Chirikov's criterion is not affected by the magnetic torque. It is further observed that in the spin-orbit phase the regular curves start disintegrating due to magnetic torque, the increase in the eccentricity and the irregular mass distribution of the satellite and this disintegration increases with the increase in ε, n and e. It has been observed that Artificial satellite's spin orbit phase space is dominated by a chaotic zone which increases further due to magnetic torque.

REFERENCES

1. Bhardwaj, R. and R. Tuli, 2005. Non-Linear Planar Oscillation of a Satellite Leading to Chaos Under the Influence of Third-body Torque. Mathematical Models and Methods for Real World Systems. Eds., Furati, Nashed, Siddiqi, Chapman & Hall/ CRC Publication, pp: 301-336.
2. Sidlichovský, M., 2005. A non-planar circular model for the 4/7 resonance. Celestial Mechanics and Dynamical Astronomy, 93: 167-185.
3. Tiscareno, M.S., 2005. Chaotic diffusion in the outer solar system and other topics. Ph. D. Thesis, pp: 151. United States-Arizona, The University of Arizona. Publication Number: AAT 3145138. DAI-B 65/09, pp: 4623.

4. Kauprianov, V.V. and I.I. Shevchenko, 2005. Rotational dynamics of planetary satellites: A survey of regular and chaotic behavior. Icarus, 176: 224-234.
5. Contopoulos, G. and K. Efstathiou, 2004. "Escapes and recurrence in a simple Hamiltonian system. Celestial Mechanics, 88: 163-183.
6. Selaru, D., V. Mioc, C. Cucu-Dumitrescu and M. Ghenescu, 2005. Chaos in Hill's generalized problem: from the solar system to black holes. Astronomische Nachrichten, 326: 356-361.
7. Carruba, V., D. Nesvorný, J.A. Burns, M. Cuk and K. Tsiganis, 2004. Chaos and the effects of planetary migration on the orbit of s/2000 S5 Kiviuq. The Astronomical J., 128: 1899-1915.
8. Bhardwaj, R. and P. Kaur, 2006. Satellite's motion under the effect of magnetic torque. Am. J. Appl. Sci., (In press).
9. Chirikov, B.V., 1969. Research concerning the theory of non-linear resonance and stochasticity. Nuclear Physics section of the Siberian Academy of Sciences, Report 267. (In Russian).