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Dynamic Response of Loads on Viscously Damped Axial Force Rayleigh Beam

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Abstract: This research examines the effects of moving loads on viscously damped axial force Rayleigh beam. The authors especially tried to find the effect of the moving mass and moving force in connection with the length of the span of a Rayleigh beam. The authors also examined the effect of the lengths of the beam and of the load. It was observed that as mass of the moving load increases the deflection along the length of the beam also increases. It was further observed that the deflection of the moving mass is greater than that of the moving force.

Key words: Viscously damped, axial force, rayleigh beam, moving force and moving mass

INTRODUCTION

From historical viewpoint, the problem of moving load back to the beginning of the nineteenth century, the time of erection of the early railway bridges. A lot of work has been done during the past years on the dynamic response of railway bridges and later of highway bridges under the influence of moving masses. A comprehensive review of work done, as a matter of fact, can be found in Fryba [6], book.

The problems of moving loads are being studied in technically advanced countries world over especially Czechoslovakia, USA, Germany, Switzerland, France and Japan to mention but a few. Both the theoretical and experimental information on the effect of moving on structures were made available. (Stokes,[18]; Krylov,[13]; Jeffcolt,[10]; Inglis,[8]; Kolousek,[12]; Bolotin,4; Steele,[17]; Knowles,[11]; Oni, [16]; Ghorashi and Esmailzadeh,[7]; Krylov,[13]; Lee,[14]; Lin,[15]; Idowu,[9]; Dada,[5]; Adetunde,[1]; Akinpelu,[3]).

The present research is an extension of the earlier research of Adetunde [2], in which the axial force is taken into consideration (Axial force beams simply means beams which do experience compression when no external force is applied, i.e., artificial creation of stresses in structure before loading.

The purpose of this research is therefore to find the effects of moving mass and moving load in connection with the length of the beam. Find the effect of the length of the load on the beam make a comparison between the deflections due to the moving mass and that of the moving force.

Under the assumption that the beam is prismatic while rotary and damping are taken into consideration.

MATHEMATICAL FORMULATION

Consider the simply supported axial force Rayleigh beam, shown in Fig. 1, of length L having a uniform cross-section with constant mass per unit length m and flexural stiffness EI. The beam is traversed by a constant load P having mass M moving at a constant velocity V, which assumed to strike a finite axial force Rayleigh beam from the left end of the beam at time t = 0 (where t is measured from the time the load enters the beam) and advancing uniformly along the beam. Before the instant, the deflection throughout the length of the beam is assumed to be zero.

The governing differential equation of motion for axial force Rayleigh beam when rotatory and damping are considered is given as:

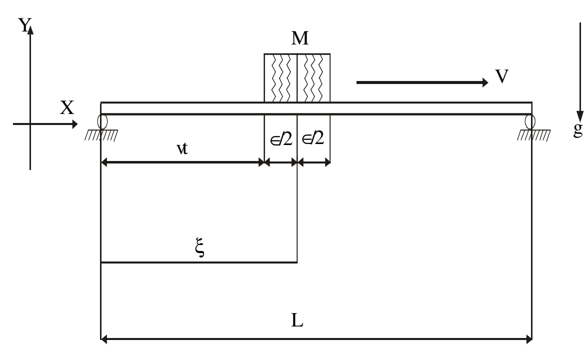

Fig 1: Mathematical Model of the problem

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$$\frac{\partial^2}{\partial x^2}\left[EI(x)\left(\frac{\partial^2 W(x,t)}{\partial x^2}+a_1\frac{\partial^3 W(x,t)}{\partial x^2\partial t}\right)\right]+m(x)\partial^2\frac{W(x,t)}{\partial t^2}$$
$$+\frac{\partial}{\partial x}\left[N\frac{\partial^2 W(x,t)}{\partial x\partial t}\right]+C(x)\frac{\partial W(x,t)}{\partial t}=P(x,t) \quad (1)$$

where, a_1 is the stiffness proportionality facto (damping complex or radius of gyration), W(x,t) is the transverse displacement response, X is the spatial coordinate, c(x) is the external damping force per unit length, N(x) is the axial force and P(x,t) is the transverse loading inertia.

The transverse load inertia takes the form described as:

$$P(x,t)=\frac{1}{\in}\left[-Mg-M\Delta\right]\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right] \quad (2)$$

$$H(x)=\begin{cases}0 & x<0\\ 1 & x>0\end{cases} \quad (3)$$

And

$$\Delta=\frac{\partial^2 W}{\partial t^2}+2V\frac{\partial^2 W}{\partial x\partial t}-\frac{V^2\partial^2 W}{\partial x^2} \quad (4)$$

where, g is the gravitational force, H is the heavyside unit function of the beam, ε is the fixed length of the load and ξ is a particular distance along the length of the beam. We employ the Dirac delta function

$$\delta(x-\xi)=\frac{1}{\in}\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi+\frac{\in}{2}\right)\right] \quad (5)$$

The mass M of the load P is not negligible but of comparable magnitude with the mass of the total beam mL. As a result of this, we consider the effect of Coriolis force (Complementary acceleration) and of centripetal force (Acceleration related to curvature R of the deflection curve) associated with the mass M of the moving load P = Mg.
Substituting Eq. (4) into Eq. (2) we have

$$P(x,t)=\frac{1}{\in}\left[-Mg-M\frac{\partial^2 W}{\partial t^2}-2MV\frac{\partial^2 W}{\partial t^2}+MV^2\frac{\partial^2 W}{\partial x^2}\right]$$
$$\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right] \quad (6)$$

OPERATIONAL SIMPLIFICATION OF THE GOVERNING EQUATION

A series solution in terms of normal modes can be sought in the form.

$$W(x,t)=\sum_{i=1}^{\infty}\phi_i(x)Y_i(t) \quad (7)$$

Where, $\phi_i(x)$'s are the shape function for the n^{th} mode of the freely Vibrating prismatic beam while $Y_i(t)$ is the corresponding modal amplitude that has to be determined. Introducing Eq. 7 into Eq. 1 and 6, we have

$$\sum_{i=1}^{\infty}m(x)\phi_i(x)\ddot{Y}_i(t)+\sum_{i=1}^{\infty}c(x)\phi_i(x)\dot{Y}_i(t)+\sum_{i=1}^{\infty}\frac{d^2}{dx^2}\left[a_1EI(x)\frac{d^2\phi_i(x)}{dx^2}\right]\dot{Y}_i(t)+\sum_{i=1}^{\infty}\frac{d}{dx}\left[N\frac{d^2\phi_i(x)}{dx}\right]\dot{Y}_i(t)$$
$$+\sum_{i=1}^{\infty}\frac{d^2}{dx^2}\left[EI(x)\frac{d^2\phi_i(x)}{dx^2}\right]Y_i(t)=\frac{1}{\in}\left[-Mg-M\sum_{i=1}^{\infty}\ddot{Y}_i(t)\phi_i(x)-2MV\sum_{i=1}^{\infty}\dot{Y}_i\frac{d\phi_i(x)}{dx}-\right. \quad (8)$$
$$\left.MV^2\sum_{i=1}^{\infty}Y_i(t)\frac{d^2\phi_i(x)}{dx^2}\right]\left[H\left(\phi-\xi+\frac{\in}{2}\right)-H\left(\phi-\xi-\frac{\in}{2}\right)\right]$$

Multiply Eq. (8) by $\phi_n(x)$ and integrate along the length of the beam and applying the two orthogonality relationships, to the Eq. (8)

$$m\sum_{i=1}^{\infty}\ddot{Y}_i(t)\int_0^L\phi_i(x)\phi_n(x)dx+C(x)\sum_{i=1}^{\infty}\dot{Y}_i(t)\int_0^L\phi_i(x)\phi_n(x)dx+\int_0^L\phi_n(x)\left[\sum_{i=1}^{\infty}\frac{d^2}{dx^2}\left[a_1EI(x)\frac{d^2\phi_i(x)}{dx^2}\right]\dot{Y}_i(t)\right]dx+$$
$$\int_0^L\phi_n(x)\left[\sum_{i=1}^{\infty}\frac{d}{dx}\left[N\frac{d\phi_i(x)}{dx}\right]\dot{Y}_i(t)\right]dx+\int_0^L\phi_n(x)\left[\sum_{i=1}^{\infty}\frac{d^2}{dx^2}\left[EI(x)\frac{d^2\phi_i(x)}{dx^2}\right]Y_i(t)\right]dx=\int_0^L\phi_n(x)\left[\frac{1}{\in}\left[-Mg-M\sum_{i=1}^{\infty}\ddot{Y}_i(t)\phi_i(x)-\right.\right. \quad (9)$$
$$\left.\left.2MV\sum_{i=1}^{\infty}\dot{Y}_i(t)\frac{d\phi_i(x)}{dx}-MV^2\sum_{i=1}^{\infty}Y_i(t)\frac{d^2\phi_i(x)}{dx^2}\right]\left[H\left(\phi-\xi+\frac{\in}{2}\right)-H\left(\phi-\xi-\frac{\in}{2}\right)\right]\right]d\theta$$

Note

$$\omega_n^2 m(x)\phi_n(x) = \frac{d^2}{dx^2}\left[EI(x)\frac{d^2\phi_n(x)}{dx^2}\right] \quad (9a)$$

$$\int_0^L \phi_n \frac{d^2}{dx^2}\left[EI(x)\frac{d^2\phi_i(x)}{dx^2}\right]dx = \omega_n^2\int_0^L \phi_n^2(x)m(x)dx \quad (9b)$$

$$\int_0^L \phi_n(x)\frac{d^2}{dx^2}\left[EI(x)\frac{d^2\phi_i(x)}{dx^2}\right]dx = 0 \text{ when } i \neq n \quad (9c)$$

$$\left.\begin{aligned}&\int_0^L \phi_n(x)\left[H\left(\phi(x)-\xi+\frac{\in}{2}\right)-H\left(\phi(x)-\xi-\frac{\in}{2}\right)\right]d\phi \\ &= \in\left[\phi_i(\xi)+\frac{\in^2}{24}\phi_i''(\xi)\right]....\end{aligned}\right] \quad (9d)$$

$$\int_0^L \phi_n(x)\phi_i(x)\left[H\left(\phi(x)-\xi+\frac{\in}{2}\right)-H\left(\phi(x)-\xi-\frac{\in}{2}\right)\right]d\phi = \in\left[\phi_i(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i''(\xi)\phi_n(\xi)+2\phi_i'(\xi)\phi_n'(\xi)+\phi_i(\xi)\phi_n''(\xi)\right]\right] \quad (9e)$$

$$\int_0^L \phi_i'(x)\phi_n(x)\left[H\left(\phi(x)-\xi+\frac{\in}{2}\right)-H\left(\phi(x)-\xi-\frac{\in}{2}\right)\right]d\phi = \in\left[\phi_i'(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i'''(\xi)\phi_n(\xi)+2\phi_i''(\xi)\phi_n'(\xi)+\phi_i'(\xi)\phi_n''(\xi)\right]\right] \quad (9f)$$

$$\int_0^L \phi_i''(x)\phi_n(x)\left[H\left(\phi(x)-\xi+\frac{\in}{2}\right)-H\left(\phi(x)-\xi-\frac{\in}{2}\right)\right]d\phi = \in\left[\phi_i''(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i^{iv}(\xi)\phi_n(\xi)+2\phi_i'''(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n''(\xi)\right]\right] \quad (9g)$$

Putting Eq. (9a-g) into Eq. (9), we have.

$$\begin{aligned}&M_n\ddot{Y}_n(t)+\sum_{i=1}^{\infty}\dot{Y}_i(t)\int_0^L\phi_n(x)\left[C_n(x)\phi_i(x)+\frac{d^2}{dx^2}\left[a_1EI(x)\frac{d^2\phi_i(x)}{dx^2}\right]dx+\omega^2NM_nY_n(t)+\omega^2M_nY_n(t)=\sum_{i=1}^{\infty}\phi_n(x)\right.\\&-Mg\left[\phi_i(\xi)+\frac{\in^2}{24}\phi_i''\right]-M\sum_{i=1}^{\infty}\ddot{Y}_i(t)\left[\phi_i(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i''(\xi)\phi_n(\xi)+2\phi_i'(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n(\xi)\right]\right]\\&-2MV\sum_{i=1}^{\infty}\dot{Y}_i(t)\left[\phi_i'(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i'''(\xi)\phi_n(\xi)+2\phi_i''(\xi)\phi_n'(\xi)+\phi_i'(\xi)\phi_n''(\xi)\right]\right]\\&-MV^2\sum_{i=1}^{\infty}Y_i(t)\left[\phi_i''(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\left[\phi_i^{iv}(\xi)\phi_n(\xi)+2\phi_i'''(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n''(\xi)\right]\right]\right.\end{aligned} \quad (10)$$

Remarks: Clearly all terms in the series in the third term of Eq. (9) go to zero except for i = n the modes are obviously uncoupled as far as the stiffness proportional damping is concerned. Coupling will be present, however, due to c(x), unless it takes on a form allowing only the term with i = n to remain in the series. This is indeed the case for mass-proportional damping, that is, if we let $c(x) = a_0m(x) = a_0m$, in which the proportionality constant a_0 has a dimension of t^{-1}, then we have

$$\begin{aligned}&M_n\ddot{Y}_n(t)+(a_0M_n+a_1M_n\omega^2)\dot{Y}_n(t)+\omega^2NM_nY_n(t)+\omega^2M_nY_n(t)=\sum_{i=1}^{\infty}\phi_n(\xi)\left\{-Mg\left[\phi_i(\xi)+\frac{\in^2}{24}\phi_i''(\xi)\right]-\right.\\&M\sum_{i=1}^{\infty}\ddot{Y}_i(t)\left[\phi_i(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i''(\xi)\phi_n(\xi)+2\phi_i'(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n(\xi)\right]\right]-\\&2MV\sum_{i=1}^{\infty}\dot{Y}_i(t)\left[\phi_i'(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i'''(\xi)\phi_n(\xi)+2\phi_i''(\xi)\phi_n'(\xi)+\phi_i'(\xi)\phi_n''(\xi)\right]\right]-\\&\left.MV^2\sum_{i=1}^{\infty}Y_i(t)\left[\phi_i''(\xi)\phi_n(\xi)+\frac{\in^2}{24}\left[\phi_i^{iv}(\xi)\phi_n(\xi)+2\phi_i'''(\xi)\phi_n'(\xi)+\phi_i''(\xi)\phi_n''(\xi)\right]\right]\right\}\end{aligned} \quad (11)$$

Introducing the damping ratio for the n^{th} mode, we have

$$\xi_n = \frac{C_n}{2M_n\omega_n} = \frac{a_0}{2\omega_n} = \frac{a_1\omega_n}{2} \tag{12}$$

Hence Eq. (11) now becomes

$$\begin{aligned}&\ddot{Y}_n(t) + 2\xi_n\omega_n\dot{Y}_n(t) + \omega^2 N\left(\frac{i\pi}{L}\right)Y_n(t) + \omega^2 Y_n(t) = \frac{M}{M_n}\sum_{i=1}^{\infty}\phi_i(x)\Bigg\{-g\Bigg[\phi_i(x) + \frac{\in^2}{24}\phi_i''(x) - \sum_{i=1}^{\infty}\ddot{Y}_i(t)\big[\phi_i(x)\phi_n(x) + \\ &\frac{\in^2}{24}\big[\phi_i''(x)\phi_n(x) + 2\phi_i'(x)\phi_n'(x) + \phi_i''(x)\phi_n(x)\big]\Bigg] - 2V\sum_{i=1}^{\infty}\dot{Y}_i(t)\Bigg[\phi_i'(x)\phi_n(x) + \frac{\in^2}{24}\big[\phi_i'''(x)\phi_n(x) \\ &+2\phi_i''(x)\phi_n'(x) + \phi_i'(x)\phi_n''(x)\big]\big] - V^2\sum_{i=1}^{\infty}Y_i(t)\Bigg[\phi_i''(x)\phi_n(x) + \frac{\in^2}{24}\big[\phi_i^{iv}(x)\phi_n(x) + \\ &+2\phi_i'''(x)\phi_n'(x) + \phi_i''(x)\phi_n''(x)\big]\Bigg]\Bigg\}\end{aligned} \tag{13}$$

SIMULATIONS

Simply supported beam: We shall now consider a simply supported beam configuration whose normalized deflection curve is given as

$$\phi_i(x) = \sqrt{\left(2/L\right)}\sin\frac{n\pi x}{L}, \quad n = 1,2,3...... \tag{14}$$

We can write

$$\omega_n = n^2\pi^2\sqrt{\left(\frac{EI}{M_n L^4}\right)}, \quad n = 1,2,3....... \tag{14a}$$

By putting Eq. (14) into the r.h.s. of Eq. (13), after a lot of simplification had been done, we finally have

$$\begin{aligned}&\ddot{Y}_n(t) + 2\xi_n\omega_n\dot{Y}_n(t) + \omega_n^2 N\left(\frac{i\pi}{L}\right)Y_i(t) + \omega_n^2 Y_i(t) = \frac{M}{M_n}\left[\frac{-1}{n\pi\xi}\sqrt{8L}\sin\frac{i\pi\xi}{2L}\sin\frac{n\pi\xi}{L} - \frac{2}{\in\pi}\sum_{i=1}^{\infty}\ddot{Y}_i(t)\left[\frac{1}{(i-n)}\mathrm{Cos}\pi\xi\frac{(i-n)}{L}\right]\right. \\ &\mathrm{Sin}\pi\xi\frac{(i-n)}{2L} + \frac{2}{\in\pi}\sum_{i=1}^{\infty}\dot{Y}_i(t)\left[\frac{1}{(i+n)}\mathrm{Cos}\pi\xi\frac{(i+n)}{L}\mathrm{Sin}\pi\xi\frac{(i+n)}{2L}\right] - \frac{2V}{\in}\sum_{i=1}^{\infty}Y_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i+n)}\mathrm{Sin}\frac{\in\pi}{2L}(i+n)\mathrm{Sin}\frac{\pi\xi}{L}(i+n)\right] \\ &-\frac{2V}{\in}\sum_{i=1}^{\infty}Y_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i-n)}\mathrm{Sin}\frac{\in\pi}{2L}(i-n)\mathrm{Sin}\frac{\pi\xi}{L}(i-n)\right] + \frac{V^2}{\in}\frac{(i\pi)}{L}\sum_{i=1}^{\infty}Y_i(t)\left[\sqrt{\frac{2}{L}}\frac{1}{(i+n)}\mathrm{Sin}\frac{\in\pi}{2L}(i+n)\mathrm{Sin}\frac{\pi\xi}{L}(i+n)\right]\end{aligned} \tag{15}$$

The above Eq. (15) is the exact governing equation of a simply supported viscously damped Rayleigh beam. We now use the finite difference method to solve the above Eq. (15) numerically. To obtain the results, we make use of central difference formula, which finally resulted into a system of equations which was in turn solved by a Visual Basic program.

NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results in both tabular and graphical forms are presented. The numerical analysis is in two folds. The first one concerns a viscously damped axial force Rayleigh beam moving mass whilst the second concerns a viscously damped axial force Rayleigh beam moving force.

The mathematical model discussed herein is related to the work done by Adetunde[2] in which the following data were used.

m (Mass per unit length of beam) = 70 Kg m^{-1}, M(Mass of the load) = 7.04, 8,10 Kg m^{-1}, g = 9.81 m/s^2
π = 22/7, L = 10 m, ξ = vt+ ε/2, ε = 0.001 m, 0.01 m, 0.1 m, v = 3.33 m/s, t = 0.5s, t = 1.0 s and t = 1.5 s

h = 0.01, I = 1.04×10^{-6} m^4, E = 2.07 X 10^{11} N/m^2, N = 0.5, $\omega_n = n^2\pi^2\sqrt{\left(\frac{EI}{M_n L^4}\right)}$, $\xi_n = 1$.

Hence, we have the followings.

Tables 1-4 and Fig. 2-5 show the variation of deflection of the beam acted upon by a moving mass and moving force.

It is observed from Fig. 2 above that as the mass of the moving load increases the deflection along the length of the beam increases.

Table 1: Effect of mass of load on deflection of the beam under moving mass

	Deflection		
x	m = 10	m = 8	m = 7
0	0	0	0
1	-1.93E-06	-1.54E-06	-1.35E-06
2	-3.45E-07	-2.76E-07	-2.42E-07
3	4.11E-06	3.29E-06	2.87E-06
4	9.78E-06	7.82E-06	6.84E-06
5	1.47E-05	1.18E-05	1.03E-05
6	1.74E-05	1.39E-05	1.22E-05
7	1.71E-05	1.37E-05	1.20E-05
8	1.36E-05	1.08E-05	9.49E-06
9	7.47E-06	5.98E-06	5.23E-06
10	-3.11E-08	-2.49E-08	-2.18E-08

Table 2: Effect of load length on deflection of the beam under moving mass

	Deflection		
x	Eps = 1	Eps = 2	Eps = 3
0	0	0	0
1	-1.93E-06	-1.92E-06	-1.88E-06
2	-3.45E-07	-3.49E-07	-3.38E-07
3	4.11E-06	4.07E-06	4.00E-06
4	9.78E-06	9.69E-06	9.53E-06
5	1.47E-05	1.46E-05	1.43E-05
6	1.74E-05	1.73E-05	1.70E-05
7	1.71E-05	1.69E-05	1.66E-05
8	1.36E-05	1.34E-05	1.32E-05
9	7.47E-06	7.39E-06	7.25E-06
10	-3.11E-08	-3.08E-08	-3.02E-08

Table 3: Effect of mass of load on deflection of beam under moving force

	Deflection		
x	m = 10	m = 8	m = 7
0	0	0	0
1	-1.90E-06	-1.50E-06	-1.30E-06
2	-3.40E-07	-2.70E-07	-2.40E-07
3	4.10E-06	3.28E-06	2.87E-06
4	9.76E-06	7.81E-06	6.83E-06
5	1.47E-05	1.17E-05	1.03E-05
6	1.74E-05	1.39E-05	1.22E-05
7	1.71E-05	1.36E-05	1.19E-05
8	1.35E-05	1.08E-05	9.48E-06
9	7.47E-06	5.97E-06	5.23E-06
10	-3.10E-08	-2.50E-08	-2.20E-08

Table 4:

	Deflection		
x	Eps = 1	Eps = 2	Eps = 3
0	0	0	0
1	-1.90E-06	-1.90E-06	-1.90E-06
2	-3.40E-07	-3.40E-07	-3.30E-07
3	4.10E-06	4.05E-06	3.97E-06
4	9.76E-06	9.64E-06	9.44E-06
5	1.47E-05	1.45E-05	1.42E-05
6	1.74E-05	1.72E-05	1.68E-05
7	1.71E-05	1.68E-05	1.65E-05
8	1.35E-05	1.34E-05	1.31E-05
9	7.47E-06	7.37E-06	7.22E-06
10	-3.10E-08	-3.10E-08	-3.00E-08

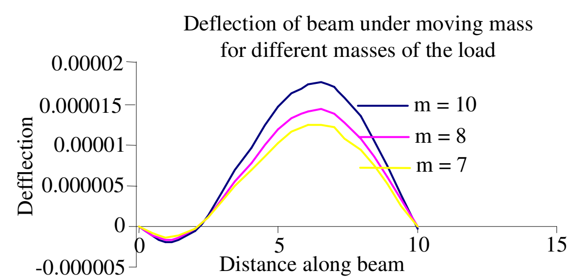

Fig. 2: Deflection of beam under moving mass for different masses of load

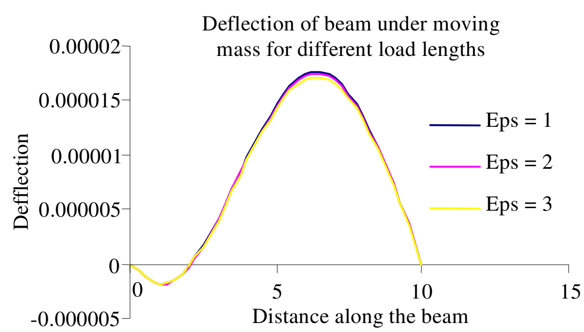

Fig. 3: Deflection of beam under moving mass for different load lengths

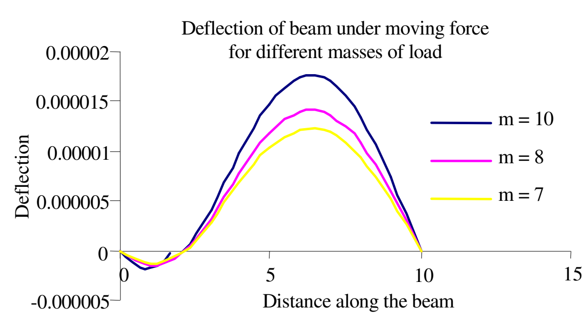

Fig. 4: Deflection of beam under moving force for different masses of load

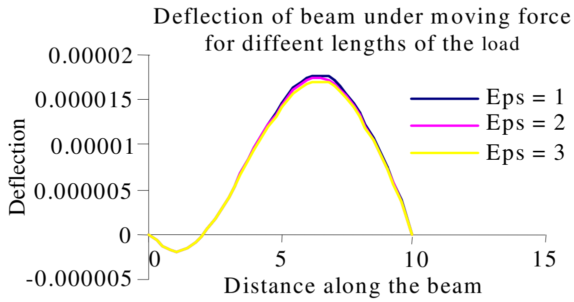

Fig. 5: Deflection of beam under moving force for different lengths of the load

It is observed from Fig. 3 above that as the length (Eps) of the moving load increases the deflection along the length of the beam increases.

It is observed from Fig. 4 above that as the mass of the moving force increases the deflection along the length of the beam increases.

It is observed from Fig. 5 above that as the length (Eps) of the moving load increases the deflection along the length of the beam increases.

SUMMARY OF RESULTS

The dynamic response of loads on viscously damped axial force Rayleigh beam was carried out.
The results obtained can be summarized as follows.

- The deflection of a viscously damped axial force Rayleigh beam under a moving mass or moving force increases with increasing mass of load
- The deflection of a viscously damped axial force Rayleigh beam under a moving mass or moving force increases with increasing span of load
- The deflection of beam due to moving mass is greater than the deflection due to moving force

CONCLUSION

The dynamic response of loads on viscously damped axial force Rayleigh beam is studied. The theory is based on orthogonal functions and the results indicate that the governing differential equation can be transformed into a series of coupled ordinary differential equations which is the solution for the corresponding moving distributed force. The resulting governing differential equation is solved by numerical approach (Finite central difference method).

In conclusion, the deflection due to moving mass is greater than that due to moving force.

REFERENCES

1 Adetunde, I.A., 2003. Dynamic analysis of Rayleigh beams carrying an added mass and traversed by uniform partially distributed moving load. Ph.D Thesis. University of Ilorin. Ilorin. Nigeria.

2 Adetunde, I.A., 2007. Dynamical behaviour of viscously damped Rayleigh beam traversed by uniform partially distributed moving loads. Res. J. Applied Sci., 2 (2): 170-176.

3 Akinpelu, F.O., 2003. The response of Euler-Bernoulli beams with an attached mass to uniform partially distributed moving load. Ph.D Thesis. University of Ilorin. Ilorin. Nigeria.

4 Bolotin, V.V., 1964. The dynamic stability of elastics Bolotin System. San Francisco Holden Day, pp: 134-141.

5 Dada, M.S., 2002. Vibration analysis of elastic Plates under uniform partially distributed loads. Ph.D Thesis. University of Ilorin, Ilorin. Nigeria.

6 Fryba, L., 1972. Vibration of solids and structures under moving loads, Groningen: Noordhoff. International Publishing Groningen, Nertherland. Vol 1 and 2. pp 1-29

7 Ghorashi, M. and E. Esmailzadeh, 1995. Vibration analysis of beams traveled by a moving mass. J. Eng., 8: 213-220.

8 Inglis, C.E., 1934. A mathematical Treatise on vibration of railway Bridges. Cambridge. Cambridge University press.

9 Idowu, S.A., 1998. The effect of added mass on the dynamic response of elastic beams traveled by concentrated moving masses. Ph.D Thesis. University of Ilorin. Ilorin, Nigeria.

10 Jeffcolt, H.H., 1929. On the Vibration of beams under the action of moving loads.Philosophy Magazine Series 7,8 No 48: 66-67.

11 Knowles, J.K., 1968. On the dynamic response of a beam to a randomly moving load. J. Applied Mechanics, 35 (1): 1-6.

12 Kolousek, V., 1961. Dynamics of Civil Engineering Structure Part I, Part II and Part III 2nd Edn., Selected Topics SNTL Prague.

13 Krylov, A.N., 1995. Mathematical collection of papers of the Academy of Sciences. Vol 61 Petersburg.

14 Lee, H.P., 1994. Dynamic response of a beam with intermediate point constraints subjected to a moving load. J. Sound and Vibration, 171: 361-368.

15 Lin, Y.H., 1996. Vibration analysis of beams traveled by uniform partially distributed moving masses. J. Sound and Vibration, 199: 667-700.

16 Oni, S.T., 1991. On the dynamic response of elastic structures to multimasses system. J. Sound and Vibration Ph.D Thesis. University of Ilorin. Ilorin. Nigeria.

17 Steele, C.R., 1971. Beams and Shells with moving loads. Int. J. Solids Stuct., 7: 1171-1198.

18 Stokes, G.G., 1849. Discussion of a differential equation relating to breaking of railways bridges. Transactions of the Cambridge Philosophical Society, 85: 707-735.