EEG Signals During Epileptic Seizure as a Semigroup of Upper Triangular Matrices

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Abstract: Problem statement: EEG signals during epileptic seizure can be presented as an algebraic structure, namely semigroup of upper triangular matrices. **Approach:** EEG signals during seizure were recorded and composed into set of matrices. They were transformed into upper triangular matrices using QR-Schur decomposition and finally as a semigroup of upper triangular matrices. **Results:** EEG signals during epileptic seizure were transformed as a semigroup of upper triangular matrices under matrix multiplication. **Conclusion:** This study described the procedure to transform signal during epileptic seizure (brainstorm) into an algebraic structure. This is the key step that will enable us to further proceed to obtain some pattern out of the seizure data in our future research.

Key words: Electroencephalography, semigroup

INTRODUCTION

Electroencephalography (EEG) is a recording of electrical activity originating from the brain. It plays an important diagnostic role in epilepsy and provides supporting evidence of a seizure disorder as well as assisting with classification of seizures. EEG has been used extensively to record the abnormal brain activity associated with epileptic seizures. It is recorded on the surface of the scalp using electrodes, thus the signal is retrievable non-invasively. Abnormal patterns such as spikes, sharp waves and, spikes and wave complexes can be recorded. The type of activity and the area of the brain that is recorded from EEG will assist the physician in prescribing the correct medication for certain type of epilepsy. Patients with epilepsy that cannot be controlled by medication will often have surgery in order to remove the damaged tissue. Thus the EEG plays an important role in localizing this tissue.

EEG analysis still relies mostly on its visual inspection. Due to the fact that visual inspection is very subjective and hardly allows any statistical analysis or standardization, several methods were proposed in order to quantify the information of the EEG. Among these, the Fourier Transform emerged as a very powerful tool capable of characterizing the frequency components of EEG signals, even reaching diagnostic importance. However, Fourier Transform has some disadvantages that limit its applicability and therefore, other methods for extracting "hidden" information from the EEG signals was necessary.

Literature review: Fuzzy Topographic Topological Mapping (FTTM) is a novel method for solving neuromagnetic inverse problem to determine the current source, i.e., epileptic foci. FTTM Version 1 has been developed to present a 3-D view of an unbounded single current source (Ahmad *et al*., 2008b; Yun and Ahmad, 2003) in one angle observation (upper of a head model). Basically, this newly developed model is a topological mapping which contains some fuzzy structures and it comprises four components linked by three different algorithms as shown in Fig. 1.

The four components are Magnetic Contour Plane (MC), Base Magnetic Plane (BM), Fuzzy Magnetic Field (FM) and Topographic Magnetic Field (TM) Fig. 1.
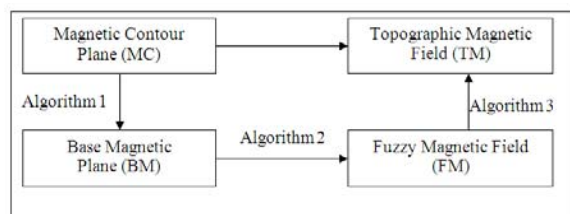

Fig. 1: FTTM version 1

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MC is actually a magnetic field on a plane above a current source with z = 0. The plane is lowered down to BM, which is a plane of the current source with z = -h. Then the entire BM is fuzzified into a Fuzzy environment (FM), where all the magnetic field readings are fuzzified. The final process is defuzzification of the fuzzified data to obtain a 3-D view of the current source (TM).

Ahmad *et al*. (2008a) has developed a new method for mapping high dimensional signal, namely EEG into a low dimensional space (MC). The whole processes of this novel model consisted three main parts. The first part was flattening the EEG where the transformation of three dimensional space into two dimensional space that involved location of sensor in patients head with EEG signal. The second part is the EEG signal was then processed using Fuzzy c-Means. The last part was to find the optimal number of cluster by using cluster validity analysis.

Ahmad's EEG coordinate system Fig. 2a is defined as:

$$C_{EEG} = \{((x, y, z), e_p): x, y, z, e_p \in R \text{ and } x^2+y^2+z^2 = r^2\}$$

where, r is the radius of a patient head. She modeled the human's head as a sphere.
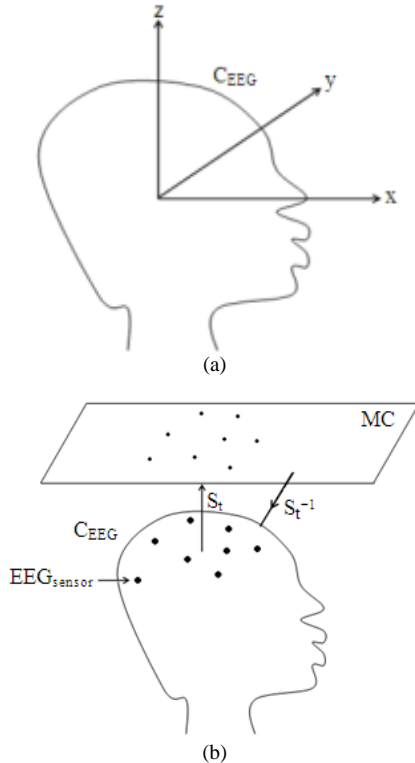

(a)

(b)

Fig. 2: (a) EEG coordinate system (b) EEG projection

Furthermore, the mapping of C_{EEG} to a plane (MC) is defined as follows.

S_t: $C_{EEG} \rightarrow MC$ Fig. 2b such that:

$$S_t \left((x, y, z), e_p\right) = \left(\frac{rx + iry}{r + z}, e_p\right) = \left(\frac{rx}{r + z}, \frac{ry}{r + z}\right)_{e_p(x,y,z)}$$

Both C_{EEG} and MC were designed and proven in (Yun and Ahmad, 2003) as 2-manifolds. Ahmad *et al*. (2008a) also had shown that S_t is a one to one function as well as being conformal. With the fact that S_t is conformal, therefore the mapping can preserve information, in particular angle and orientation of the surface and EEG signal recorded from the surface of high dimensional into a low dimensional spaces; i.e. mapping EEG signal into a plane. Then, Ahmad *et al*. (2008a) implements this technique followed by clustering on the real time EEG data obtained from patients who suffer from epileptic seizure.

The signals were digitized at 256 samples per second using Nicolet One EEG software. The average potential difference was calculated from the 256 samples of raw data at every second. Similarly to the position of electrodes, the EEG signal was also preserved during this new method. Subsequently, every single second of the particular average potential difference was stored into a file which contains the position of electrode on MC plane.

MATERIALS AND METHODS

In our study, we rewrite MC in terms of square matrices. Therefore every single second of the particular average potential difference was stored into a square matrix which contains the position of electrode on MC plane. Thus Magnetic Contour Plane became a set of (n×n) square matrices defined as:

$$MC_n = \left\{\left[\beta_{ij}(z)_t\right]_{n \times n} : i, j \in \mathbf{Z}^+, \beta_{ij}(z)_t \in \mathbf{R}\right\}$$

where, $\beta_{ij}(z)_t$ is a potential difference reading of EEG signals from a particular ij sensor at time t.

Semigroup of MC_n: We are going to show that the nonempty set of square matrices (EEG signals) satisfies all the axioms of a semigroup given in (Whitelaw, 1978) under matrix multiplication. In other words, we are going to show that:

- $MC_n = \left\{\left[\beta_{ij}(z)_t\right]_{n \times n} : i, j \in \mathbf{Z}^+, \beta_{ij}(z)_t \in \mathbf{R}\right\}$ is closed with respect to matrix multiplication
- matrix multiplication on MC_n is associative

Theorem 1: The set of (n×n) square matrices MC_n is a semigroup under matrix multiplication.

Proof: Firstly, let us show that MC_n is closed with respect to matrix multiplication. We pick:

$$A = \begin{pmatrix} \beta 1_{1,1} & \cdots & \beta 1_{1,n} \\ \vdots & \ddots & \vdots \\ \beta 1_{n,1} & \cdots & \beta 1_{n,n} \end{pmatrix} \text{ and } B = \begin{pmatrix} \beta 2_{1,1} & \cdots & \beta 2_{1,n} \\ \vdots & \ddots & \vdots \\ \beta 2_{n,1} & \cdots & \beta 2_{n,n} \end{pmatrix} \in MC_n$$

Then:

$$AB = \begin{pmatrix} \beta 1_{1,1} & \cdots & \beta 1_{1,n} \\ \vdots & \ddots & \vdots \\ \beta 1_{n,1} & \cdots & \beta 1_{n,n} \end{pmatrix} \begin{pmatrix} \beta 2_{1,1} & \cdots & \beta 2_{1,n} \\ \vdots & \ddots & \vdots \\ \beta 2_{n,1} & \cdots & \beta 2_{n,n} \end{pmatrix}$$

$$= \left(\sum_{j=1}^{n} \beta 1_{i,j} \beta 2_{j,k} \right)_{(nn)}$$

Notice we go across the i-th row of the first matrix and down the k-th column of the second matrix to obtain the entry in position (i, k):

$$i \begin{pmatrix} & \vdots & \\ \beta 1_{i,1} & \beta 1_{i,2} \cdots & \beta 1_{i,n} \\ & \vdots & \end{pmatrix} \begin{pmatrix} & \beta 2_{1,k} & \\ \cdots & \vdots & \cdots \\ & \beta 2_{n,k} & \end{pmatrix} = i \begin{pmatrix} & \vdots & \\ \cdots & \sum_{j=1}^{n} \beta 1_{i,j} \beta 2_{j,k} & \cdots \\ & \vdots & \end{pmatrix}$$

Now $\beta 1_{i,j}, \beta 2_{j,k} \in \square$ for a particular time $t \in \mathbf{R}^+$ and without loss of generality, $\beta 1_{i,j} \beta 2_{j,k} \in \mathbf{R}$ for some time $t \in \mathbf{R}^+$, thus $\sum_{j=1}^{n} \beta 1_{i,j} \beta 2_{j,k} \in \square$. Since A, B $\in MC_n$ are arbitrary, therefore $AB \in MC_n$ and hence MC_n is closed with respect to matrix multiplication.

Secondly, let us show that matrix multiplication on MC_n is associative. Pick:

$$A = \left[\beta 1_{i,j} \right]_{(nn)}, B = \left[\beta 2_{j,k} \right]_{(nn)}, C = \left[\beta 3_{k,l} \right]_{(nn)}$$

Then:

$$AB = \left(\sum_{j=1}^{n} \beta 1_{i,j} \beta 2_{j,k} \right)_{(nn)}$$

$$(AB)C = \left(\sum_{k=1}^{n} (\sum_{j=1}^{n} \beta 1_{i,j} \beta 2_{j,k}) \beta 3_{k,l} \right)_{(nn)}$$

$$= \left(\sum_{k=1}^{n} \sum_{j=1}^{n} \beta 1_{i,j} \beta 2_{j,k} \beta 3_{k,l} \right)_{(nn)}$$

$$BC = \left(\sum_{k=1}^{n} \beta 2_{j,k} \beta 3_{k,l} \right)_{(nn)}$$

$$A(BC) = \left(\sum_{j=1}^{n} \beta 1_{i,j} (\sum_{k=1}^{n} \beta 2_{j,k} \beta 3_{k,l}) \right)_{(nn)}$$

$$= \left(\sum_{j=1}^{n} \sum_{k=1}^{n} \beta 1_{i,j} \beta 2_{j,k} \beta 3_{k,l} \right)_{(nn)}$$

Since:

$$\sum_{k=1}^{n} \sum_{j=1}^{n} \beta 1_{i,j} \beta 2_{j,k} \beta 3_{k,l} = \sum_{j=1}^{n} \sum_{k=1}^{n} \beta 1_{i,j} \beta 2_{j,k} \beta 3_{k,l}, \quad \text{we have}$$

(AB) C = A (BC). The associativity of MC_n reveals that historical event is preserved in time (Nehaniv and Dautenhahn, 1998). It means that the property of time is actually embedded in MC_n.

We have shown that:

- $MC_n = \left\{ \left[\beta_{ij}(z)_t \right]_{n \times n} : i, j \in \mathbf{Z}^+, \beta_{ij}(z)_t \in \mathbf{R} \right\}$ is closed with respect to matrix multiplication
- matrix multiplication on MC_n is associative

In other words, Magnetic Contour plane (MC) is a semigroup of square matrices under matrix multiplication.

Triangularization of MC_n: There are many methods to triangularize a matrix $\left[\beta_{ij}(z)_t \right]_{n \times n}$ in MC_n. For example, Elementary, Householder and Givens matrices can be used to achieve important matrix factorization such as LU and its variants, QR and reduction to a Hessenberg form of a matrix.

Since our main purpose in this study is to prove MC_n as a semigroup of (n×n) upper triangular matrices $MC_n"$ and every element in $MC_n"$ as a block matrix then we will choose the Schur decomposition to decompose our square matrix $\left[\beta_{ij}(z)_t \right]_{n \times n}$ in MC_n.

If the matrix A is real as in our case, then the real Schur Decomposition is computed, in which the matrix Q is orthogonal and U is block upper triangular with blocks of size at most 2×2 along the diagonal:

$$Q^T A Q = U$$

Theorem 2 (Datta, 1995): Let A be an n×n real matrix. Then there exists an n×n orthogonal matrix Q such that:

$$Q^T A Q = R = \begin{pmatrix} R_{11} & R_{12} & \cdots & R_{1k} \\ 0 & R_{22} & \cdots & R_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{kk} \end{pmatrix}$$

where, each R_{ii} is either a scalar or a 2×2 matrix. The scalar diagonal entries correspond to real eigenvalues and 2×2 matrices on the diagonal correspond to complex conjugate eigenvalues.

Remark: The matrix R in theorem 2 is known as the real Schur form of A.

Before we present the basic QR iteration method for computing the real Schur form of A we will present the QR factorization of A using Householder matrix.

Theorem 3 (Datta, 1995): Given an n×n matrix A, there exists an orthogonal matrix Q and an upper triangular matrix R such that:

$$A = QR$$

The matrix Q can be written as $Q = H_1 H_2 \ldots H_{n-1}$ where each H_i is a Householder matrix.

Definition 4 (Datta, 1995): A matrix of the form:

$$H = I - \frac{2uu^T}{u^T u}$$

where, u is a nonzero vector, is called a Householder matrix.

Basic QR iteration (Datta, 1995): We first present the basic QR iteration method:

Set $A_0 = A$

Now compute a sequence of matrices (A_k) defined by:
$A_0 = Q_0 R_0$ (Using theorem 3)

$$A_1 = R_0 Q_0 = Q_1 R_1$$
$$A_2 = R_1 Q_1 = Q_2 R_2$$

In general:

$$A_K = Q_K R_K = R_{K-1} Q_{K-1} \quad k = 1, 2\ldots$$

The matrices in the sequence $\{A_k\}$ have a very interesting property. Each matrix in the sequence is orthogonally similar to the previous one and is, therefore, orthogonally similar to the original matrix. For example:

$$A_1 = R_0 Q_0 = Q_0^T A_0 Q_0 \ \text{(Because } Q_0^T A_0 = R_0 \text{)}$$
$$A_2 = R_1 Q_1 = Q_1^T A_1 Q_1$$

Thus, A_1 is orthogonally similar to A and A_2 is orthogonally similar to A_1. Therefore, A_2 is orthogonally similar to A, as the following computation shows:

$$A_2 = Q_1^T A_1 Q_1 = Q_1^T \left(Q_0^T A_0 Q_0 \right) Q_1 = \left(Q_0 Q_1 \right)^T A_0 \left(Q_0 Q_1 \right)$$

Because each matrix is orthogonally similar to the original matrix A therefore it has the same eigenvalues as A, then the sequence $\{A_K\}$ converges to a triangular matrix.

Theorem 5: The set of (n×n) square matrices MC_n, where:

$$MC_n = \left\{ \left[\beta_{ij}(z)_t \right]_{n \times n} : i, j \in \mathbf{Z}^+, \beta_{ij}(z)_t \in \mathbf{R} \right\}$$

can be transformed to the set of (n×n) upper triangular matrices MC_n'', where:

$$MC_n'' = \left\{ \begin{array}{l} \left[\beta_{ij}(z)_t \right]_{n \times n} : \beta_{ij}(z)_t = 0 \, \forall 1 \leq j < i \leq n, \\ \qquad\qquad\qquad i, j \in \mathbf{Z}^+, \beta_{ij}(z)_t \in \mathbf{R} \end{array} \right\}$$

Proof: Let

$$MC_n = \left\{ \left[\beta_{ij}(z)_t \right]_{n \times n} : i, j \in \mathbf{Z}^+, \beta_{ij}(z)_t \in \mathbf{R} \right\}$$

By using theorem 3 and the basic QR iteration we obtain the form MC_n'' which is the real Schur form of MC_n.

Semigroup of MC_n": We are going to show that the set of upper triangular matrices MC_n" satisfies all the axioms of a semigroup given in (Whitelaw, 1978) under matrix multiplication. In other words, we are going to show that

- $MC_n" = \left\{ \begin{array}{l} \left[\beta_{ij}(z)_t\right]_{n \times n} : \beta_{ij}(z)_t = 0 \, \forall 1 \leq j < i \leq n, \\ \qquad\qquad i,j \in \mathbf{Z}^+, \beta_{ij}(z)_t \in \mathbf{R} \end{array} \right\}$ is

  closed with respect to matrix multiplication
- Matrix multiplication on MC_n" is associative

Theorem 6: The set of (n×n) upper triangular matrices MC_n" is a semigroup under matrix multiplication.

Proof: Using similar proof as given in theorem 1, we arrive at the proof of our theorem.

RESULTS

In this study, we have shown that Magnetic Contour Plane of Fuzzy Topographic Topological Mapping (FTTM) which contains the EEG signals during epileptic seizure can be transformed into a semigroup of upper triangular matrices under matrix multiplication.

DISCUSSION

We have shown that the EEG signals during epileptic seizure can be viewed as a semigroup of upper triangular matrices under matrix multiplication. We demonstrated the possibility of transforming from one structure to the other, namely from topological to algebraic structure of EEG signals during seizure.

CONCLUSION

This research will enable us to proceed further in identifying characteristics of EEG signals during epileptic seizure.

ACKNOWLEDGEMENT

REFERENCES

Ahmad, T., R.A. Fairuz, F. Zakaria, and H. Isa, 2008a. Selection of a subset of EEG channels of epileptic patient during seizure using PCA. Proceeding of the 7th WSEAS International Conference on Signal Processing, Robotics and Automation, Advanced Topics on Signal Processing, Robotics and Automation, Feb. 20-22, University of Cambridge, UK., pp: 270-273.

Ahmad, T., R.S. Ahmad, W.E.Z.W.A. Rahman, L.L. Yun and F. Zakaria, 2008b. Fuzzy topographic topological mapping for localization simulated multiple current sources of MEG. J. Interdiscip. Math., 11: 381-393. http://www.connectjournals.com/jim

Datta, B.N., 1995. Numerical Linear Algebra and Applications. 1st Edn., Brooks/Cole Pub., ISBN: 13: 9780534174668, pp: 680.

Nehaniv, C. and K. Dautenhahn, 1998. Embodiment and memories-algebras of time and history for autobiographic agents. Proceeding of the 14th European Meeting on Cybernetics and Systems, Apr. 14-17, Austrian Society for Cybernetic Studies, Vienna, Austria, pp: 651-656.

Whitelaw, T.A., 1978. Introduction to Abstract Algebra. 2nd Edn., Blackie, Glasgow, ISBN: 0216904889, pp: 166.

Yun, L.L. and T. Ahmad, 2003. Construction of the sphere to elipsoid homeomorfisma. Math, 19: 121-138. http://www.fs.utm.my/matematika/images/stories/matematika/200319205.pdf