Stabilization of a Nonlinear Delay System

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Abstract: Problem statement: The analysis and control of delayed systems are becoming more and more research topics in progress. This is mainly due to the fact that the delay is frequently encountered in technological systems. This can affect their significantly operations. Most control command laws are based on current digital computers and delays are intrinsic to the process or in the control loop caused by the transmission time control sequences, or computing time. The delay may affect one or more states of the considered system. It may also affect the establishment of the command. Several studies have investigated the stability of delay systems under the assumption that the delay is a variable phenomenon; such variation is considered to be bounded or limited to facilitate analysis of the system. In this study we propose a modelling of delayed system by using the multimodels and switched system theory. The analysis of stability is based on the use of second Lyapunov method. The issued stability conditions are expressed as Bilinear Matrix Inequalities impossible to resolve. That's why we propose the same original relaxations to come over this difficulty, an example of induction machine is given to illustrate over approach. **Approach:** We propose to use the control theory developed for switched systems to synthesis a control laws for the stabilisation of delays system. **Results:** We stabilize the induction machine around many operating points despite the non linearities. **Conclusion:** The developed method is less conservative and less pessimistic than the used classical methods.

Key words: Delay systems, lyapunov method, switching system, Linear Matrix Inequalities (LMI), Bilinear Matrix Inequalities (BMI)

INTRODUCTION

Research in the field of systems controlled via computer networks is growing because of the expansion of computer networks and the development of more robust engines. The economic gains of remote control are more interesting and attract many manufactories.

The analysis and synthesis of delay systems are becoming more and more research topics in progress. This is mainly due to the fact that the delay is frequently encountered in technological systems and can affect their operations significantly. Most are based on current digital computers and delays may occur intrinsically to the process or in the control loop caused by the transmission time control sequences, or computing time. The delay may affect one or more states of the considered system. It may also affect the establishment of the output.

Several studies have modeled the linear systems with delays by differential equations covering both the present and the past states of the system, assuming that the derivative of the vector of states can be explained at every time t.

Other studies consider delay systems as nonlinear and no stationary (Fridman and Shaked, 2002) (Cloosterman *et al.*, 2007) with parameters varying with time or depending on the state of the system. The representation of such variation may be continuous or piecewise continuous (Ariba and Gouaisbaut, 2007).

Approaches have emerged for the analysis of delay systems based on the nature of the delay itself. We can mention: (Zhu and Hu, 2009)which assumes that the delay is constant over time. The advantage of such an approach is the reduction of the order of the models obtained and the relative ease to design controllers the whole time the assumption of constant delay is physically questionable.

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Several studies have investigated the stability of delay systems under the assumption that the delay is a variable phenomenon (Gouaisbaut and Peaucelle, 2009; Benchohra *et al.*, 2007) such a change is seen as bounded or limited to facilitate system analysis (Hetel *et al.*, 2007).

The research is carried out to ensure the stability of delay systems are usually around an equilibrium point of the system using, the method of Lyapunov, (Hu *et al.*, 1998) is a temporal method which allows it studying mathematically complex large system dimensions (Migloire, 2004; Labit *et al.*, 2007).

Many studies have aimed at reducing the wide variety of problems of synthesis or analysis in convex optimization problems involving LMI, (Henrion *et al.*, 1999; Teixeira *et al.*, 2003). The problems of control and observation are expressed as Bilinear Matrix Inequalities (BMI) where the resolution of such constraints can have several local solutions, but there are algorithms for local and global optimizations where BMI may lead to a LMI by changing or elimination of well-defined variables (Apkarian and Tuan, 2000).

In Part I of this article, we present an approach for the synthesis of a controller for delayed discrete systems. The delay is considered type variable in time and multiple of the sampling period of the system. In Part II, relaxations are introduced to the transformation of BMI in LMI reaccredit worthy by the current numerical solvers (Matlab© or Scilab ©). An illustrative example is given in part II and consists of stabilization of asynchronous machine driven by speed variator via computers network. This example assumes that the delay is due the network. The first part is devoted to the modeling of delay systems. The second part presents the study of the stability of this type of system, which is based on the Lyapunov second method and consequently the relaxation techniques of linear matrix inequalities. The third part is devoted to modeling the induction motor and the study of the stability of the model affected by delays in the order (Kechiche *et al.*, 2011).

MATERIALS AND METHODS

Delay systems:
Definition: Delay systems are systems characterized by equations with delays that are introduced to model phenomena in which there is a lag between the action on the system and the system's response to this action. The delay may affect several elements. The delay in the state is written in the form given by Eq. 1:

$$x(t) = ax(t - \tau) + bu(t) \tag{1}$$

The delay in the order, written in the form Eq. 2:

$$x(t) = ax(t) + bu(t - \tau) \tag{2}$$

The delay in the output is written as Eq. 3:

$$y(t - \tau) = cx(t) + du(t) \tag{3}$$

Several models have been associated with delay systems and it appears that several types of systems depending on the nature of the delay.

Stability of delay systems: The stability of linear systems with time-invariant delays is usually studied using the Lyapunov Krasovskii method; this method is very conservative because it generates a lotof LMI conditions difficult to satisfy all. Another approach similar to the approach of Lyapunov Krasovskii has distinguished itself in recent years this is the approach of switched systems (Gouaisbaut and Peaucelle, 2006; Gouaisbaut, 2005).

Delay system to a switching system: The switched system can be described as follows Eq. 4:

$$x(k+1) = \sum_{i=1}^{N}\mu_i(k)A_i(k)x(k) + \sum_{i=1}^{N}\mu_i(k)B_i(k)u(k) \tag{4}$$

where, the parameters $\mu_i(k)$ replace the switching law such as $\Sigma \mu i = 1$.

The return status is written as Eq. 5:

$$u(k) = \sum_{i=1}^{N}\mu_i(k)K_i(k)x(k) \tag{5}$$

The closed loop system is given by the following Eq. 6:

$$x(k+1) = \sum_{i=1}^{N}\mu_i(k)(A_i + B_i k_i)x(k) \tag{6}$$

The formulation of the system with the presence of delays in the control u (k) = K_i x (k-τ) is described by switched systems approach that requires an augmented state vector Eq. 7:

$$z(k)) = [x(k).....x(k - \tau_1)]^T \tag{7}$$

The formulation of (4) as a switching system is as follows:

$$z(k+1) = \overline{A}_{\varphi(k)} z(k)$$

where, the matrix $\overline{A}_\varphi(k) = \overline{A}_i$ for $i \in I = \{1, \ldots \tau_1\}$ and:

$$\overline{A}_I = \begin{bmatrix} A_d & 0 & 0 & -B_d K & 0 & .. & 0 \\ I & 0 & . & & . & . & 0 \\ 0 & I & 0 & & . & . & 0 \\ . & 0 & .. & & . & . & . \\ 0 & . & . & & . & I & 0 \end{bmatrix}$$

A_d block is on the $(i+1)^{\text{éme}}$ th column of A_i.

After this transformation, system stability is proved using Lyapunov functions, poly quadratic Eq. 8 (Daafouz and Bernussou, 2001):

$$v(k) = z^T \overline{P}_\varphi(k) z(k) \tag{8}$$

where, the matrixes $\overline{P}_1, \overline{P}_2 \ldots \overline{P}_{\tau 1}$ are positive definite matrices. The system is asymptotically stable if the matrixes $\overline{P}_i, \forall i = 1 \ldots \tau_1$ check the satisfies following linear matrix inequality Eq. 9:

$$\begin{bmatrix} \overline{P}_i & \overline{P}_i^T \overline{P}_j \\ \overline{P}_j \overline{P}_i & \overline{P}_j \end{bmatrix} > 0, \forall (i,j) \in I*I \tag{9}$$

The previous system (8) is stable if there exists a state feedback gain K such that the closed loop system for any bounded delay is stable. We use the equivalent condition to the condition of Lyapunov Eq. 10:

$$\begin{bmatrix} G_i + G_i^t - S & G_i^t A_i^t \\ A_i G_i & S \end{bmatrix} > 0 \tag{10}$$

For a delay $\tau = i$, let us apply the method of switched system, the state equation becomes Eq. 11:

$$x(k+1) = Ax(k) + BKx(k-i) \tag{11}$$

Equation 12 can be rewritten as follows:

$$\begin{bmatrix} x(k+1) \\ x(k) \\ . \\ . \\ . \\ x(k-(i-1)) \end{bmatrix} = \begin{bmatrix} A_d & . & . & B_d K & . \\ I & 0 & . & . & 0 \\ 0 & I & 0 & . & . \\ 0 & . & I & 0 & . \\ . & . & . & I & . \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \\ . \\ . \\ x(k-i) \end{bmatrix} \tag{12}$$

The block $B_d k$ is in column i, the switching system adequate representation is as follows given by Eq. 13:

$$z(k+1) = A_i z(k)$$

$$\text{with } A_i = \begin{bmatrix} A_d & . & . & B_d K & . \\ I & 0 & . & . & 0 \\ 0 & I & 0 & . & . \\ 0 & . & I & 0 & . \\ . & . & . & I & . \end{bmatrix} \tag{13}$$

The LMI condition for $\tau = i$ is Eq. 14:

$$\begin{bmatrix} G_i + G_i^t - S & G_i^t A_i^t \\ A_i G_i & S \end{bmatrix} > 0 \tag{14}$$

Whether $G_i = \begin{bmatrix} g_1 & g_2 & . & . & g_{i+1} \\ . & . & & & \\ . & & . & & \\ g_{i+1} & . & & . & \end{bmatrix}$ LMI condition

becomes Eq. 15:

$$\begin{bmatrix} G_i + G_i^t - S & G_i^t A_i^t \\ A_i G_i & S \end{bmatrix} > 0 \tag{15}$$

With:

$$A_i G_i = $$
$$\begin{bmatrix} A_d g_1 & A_d g_3 - B_d K g_4 & . & . & A_d g_{i+1} - B_d K g_{i+1} \\ g_1 & g_2 & . & . & g_i + 1 \\ . & & & & \\ . & & & & \\ g_1 + 1 & . & & . & . & . \end{bmatrix}$$

The condition (15) is a BMI unsolvable by current solvers it is essential to introduce relaxations to move to LMI conditions easy to solve it Eq. 16 and 17:

$$g_i = 0 \forall i \in [1, i] \tag{16}$$

$$K_i = R_{i+1} * g_{i+1} \quad R_{i+1} = K * g_{i+1} \tag{17}$$

Illustrative example:
Consider the following discrete system Eq. 18:

$$\begin{cases} \begin{bmatrix} = 0.99964 & -2.998e-005 \\ 7.495e-005 & 0.999 \end{bmatrix} x + \\ \begin{bmatrix} 0.0002 \\ 7.497e-0090 \end{bmatrix} u \\ \qquad y = \begin{bmatrix} 0 & 1 \end{bmatrix} x \end{cases} \tag{18}$$

The matrices of the system state switched to a delay $\tau = 3$ are written as follows Eq. 19:

$$z(k+1) = A_3 z(k)$$
$$\text{with } A_3 = \begin{bmatrix} A_d & 0 & 0 & -B_d K \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \tag{19}$$

Provider adequate Lyapunov condition is Eq. 20:

$$\begin{bmatrix} G_3 + G_3^t - S & G_3^t A_3^t \\ A_3 G_3 & S \end{bmatrix} > 0 \tag{20}$$

We set $G_3 = \begin{bmatrix} g_1 & g_2 & g_3 & g_4 \\ g_4 & g_6 & g_7 & g_8 \\ g_9 & g_{10} & g_{11} & g_{12} \\ g_{13} & g_{14} & g_{15} & g_{16} \end{bmatrix}$ the BMI condition for

τ is Eq. 21:

$$\begin{bmatrix} G_3 + G_3^t - S & (*)^t \\ (*) & S \end{bmatrix} > 0 \tag{21}$$

With:

$$(*) =$$
$$\begin{bmatrix} A_d g_1 - & A_d g_2 - & A_d g_3 - & A_d g_4 - \\ B_d K g_{13} & B_d K g_{14} & B_d K g_{15} & B_d K g_{16} \\ g_1 & g_2 & g_3 & g_4 \\ g_5 & g_6 & g_7 & g_8 \\ g_9 & g_{10} & g_{11} & g_{12} \end{bmatrix}$$

Relaxations are Eq. 22:

$R_{13} = K g_{13}$, $R_{14} = K g_{14}$, $R_{15} = K g_{15}$,
$R_{16} = K g_{16}$ et $R_{13} = R_{14} = R_{15} = 0$ (22)

The introduction of these relaxations can transform BMI (22) in LMI. We find that the system is stable for a gain of state feedback Eq. 23:

$$k_3 = 1.0e - 004 *$$
$$\begin{bmatrix} -0.1453 & 0.0871 & -0.0510 & -0.0512 & -0.0511 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

For a delay $\in$ [i, j] the system is poly-quadratically stable if the following LMI conditions are satisfied Eq. 24:

Condition for $\tau = i$

$$\begin{bmatrix} G_i + G_i^t - S_i & G_i^t A_i^t \\ A_i G_i & S_i \end{bmatrix} > 0 \tag{24}$$

For a switching from subsystem i to subsystem j Eq. 25:

$$\begin{bmatrix} G_i + G_i^t - S_i & G_i^t A_i^t \\ A_i G_i & S_j \end{bmatrix} > 0 \tag{25}$$

Condition for $\tau = j$ Eq. 26:

$$\begin{bmatrix} G_j + G_j^t - S_j & G_j^t A_j^t \\ A_j G_j & S_j \end{bmatrix} > 0 \tag{26}$$

For a switching from subsystem j to subsystem I Eq. 27:

$$\begin{bmatrix} G_i + G_i^t - S_j & G_i^t A_i^t \\ A_i G_i & S_i \end{bmatrix} > 0 \tag{27}$$

If the delay $\tau = i <= j$, we have

$$A_i = \begin{bmatrix} A_d & . & . & -B_d K & 0. \\ I & 0 & . & . & 0 \\ 0 & I & 0 & . & . \\ 0 & . & I & 0 & . \\ . & . & . & I & . \end{bmatrix} \; B_d \text{ k block is in the column}$$

i.

We increase the order of the matrix or elements zero columns' until we reach the same order as A_j (j +1)* (j +1):

$$A_j = \begin{bmatrix} A_d & . & . & -B_d K & 0 \\ I & 0 & . & . & 0 \\ 0 & I & 0 & . & . \\ 0 & . & I & 0 & . \\ . & . & . & I & . \end{bmatrix}$$

B_d k block is in the jth column of a dimension (j+1 * j +1) whether:

$$G_i = \begin{bmatrix} g_{11} & g_{12} & . & . & g_{i+1} \\ . & . & & & \\ . & & . & & \\ g_{i+1} & . & & . & \end{bmatrix}$$

$$G_j = \begin{bmatrix} g_{21} & g_{22} & \cdot & \cdot & g_{j+1} \\ \cdot & \cdot & & & \\ \cdot & & \cdot & & \\ g_{j+1} & \cdot & & \cdot & \end{bmatrix}$$

BMI conditions become Eq. 28-31:

$$\begin{bmatrix} \begin{bmatrix} & G_i + G_i^t - S_i & & G_i^t A_i^t \\ A_d g_{11} & \begin{matrix} A_d g_{13} \\ -B_d K g_{14} \end{matrix} & \cdot\; \cdot & \begin{matrix} A_d g_{i+1} \\ -B_d K g_{i+1} \end{matrix} & \\ g_{11} & g_{12} & \cdot\; \cdot & g_{i+1} & \cdot \\ \cdot & & & & \\ \cdot & & & & \\ g_{i+1} & \cdot & \cdot\; \cdot & \cdot & \end{bmatrix} & S_i \end{bmatrix} > 0 \qquad (28)$$

$$\begin{bmatrix} \begin{bmatrix} & G_i + G_i^t - S_i & & G_i^t A_i^t \\ A_d g_{11} & \begin{matrix} A_d g_{13} \\ -B_d K g_{14} \end{matrix} & \cdot\; \cdot & \begin{matrix} A_d g_{i+1} \\ -B_d K g_{i+1} \end{matrix} & \\ g_{11} & g_{12} & \cdot\; \cdot & g_{i+1} & \cdot \\ \cdot & & & & \\ \cdot & & & & \\ g_{i+1} & \cdot & \cdot\; \cdot & \cdot & \end{bmatrix} & S_j \end{bmatrix} > 0 \qquad (29)$$

$$\begin{bmatrix} \begin{bmatrix} & G_j + G_j^t - S_j & & G_j^t A_j^t \\ A_d g_{21} & \begin{matrix} A_d g_{23} \\ -B_d K g_{14} \end{matrix} & \cdot\; \cdot & \begin{matrix} A_d g_{i+1} \\ -B_d K g_{j+1} \end{matrix} & \\ g_{21} & g_{22} & \cdot\; \cdot & g_{j+1} & \cdot \\ \cdot & & & & \\ \cdot & & & & \\ g_{j+1} & \cdot & \cdot\; \cdot & \cdot & \end{bmatrix} & S_j \end{bmatrix} > 0 \qquad (30)$$

$$\begin{bmatrix} \begin{bmatrix} & G_j + G_j^t - S_j & & G_j^t A_j^t \\ A_d g_{21} & \begin{matrix} A_d g_{23} \\ -B_d K g_{14} \end{matrix} & \cdot\; \cdot & \begin{matrix} A_d g_{i+1} \\ -B_d K g_{j+1} \end{matrix} & \\ g_{21} & g_{22} & \cdot\; \cdot & g_{j+1} & \cdot \\ \cdot & & & & \\ \cdot & & & & \\ g_{j+1} & \cdot & \cdot\; \cdot & \cdot & \end{bmatrix} & S_i \end{bmatrix} > 0 \qquad (31)$$

The analysis of these conditions has become difficult as they are of BMI, we introduce the following relaxations Eq. 32-34:

$$B_d K_{gi} = 0 \,\forall\, I \in [1, i\text{-}1] \qquad (32)$$

$$B_d K_{gj} = 0 \,\forall\, j \in [1, i\text{-}1] \qquad (33)$$

$$K_i = R_{i+1} * g_{i+1}^{-1} \quad R_{i+1} = K * g_{i+1} \qquad (34)$$

Other relaxations can be introduced as $S_i = S_j$, this optimization is local. The advantage of this hypothesis is the reduction of the number of LMI constraints Eq. 35:

$$\begin{bmatrix} \begin{bmatrix} & G_1 + G_1^t - S_1 & & (*)^t \\ A_d g_{11} & A_d g_{21} & A_d g_{31} & \\ -B_d K g_{41} & -B_d K g_{51} & -B_d K g_{61} & \\ g_{11} & g_{21} & g_{31} & \cdot \\ g_{41} & g_{51} & g_{61} & \end{bmatrix} & S_1 \end{bmatrix} > 0 \qquad (35)$$

Illustrative example: Consider the same example above, the delay τ between switches 1 and 2. BMI conditions are Eq. 36-38:

$$\begin{bmatrix} \begin{bmatrix} & G_2 + G_2^t - S_2 & & (*)^t \\ A_d g_{12} & A_d g_{22} & A_d g_{32} & \\ -B_d K g_{72} & -B_d K g_{82} & -B_d K g_{92} & \\ g_{12} & g_{22} & g_{31} & \cdot \\ g_{42} & g_{52} & g_{61} & \end{bmatrix} & S_2 \end{bmatrix} > 0 \qquad (36)$$

$$\begin{bmatrix} \begin{bmatrix} & G_1 + G_1^t - S_1 & & (*)^t \\ A_d g_{11} & A_d g_{21} & A_d g_{31} & \\ -B_d K g_{41} & -B_d K g_{51} & -B_d K g_{61} & \\ g_{11} & g_{21} & g_{31} & \cdot \\ g_{41} & g_{51} & g_{61} & \end{bmatrix} & S_2 \end{bmatrix} > 0 \qquad (37)$$

$$\begin{bmatrix} \begin{bmatrix} & G_2 + G_2^t - S_2 & & (*)^t \\ A_d g_{12} - & A_d g_{22} - & A_d g_{31} - & \\ B_d K g_{72} & B_d K g_{51} & B_d K g_{61} & \\ g_{12} & g_{22} & g_{32} & \cdot \\ g_{42} & g_{52} & g_{61} & \end{bmatrix} & S_1 \end{bmatrix} > 0 \qquad (38)$$

The relaxations are proposed to solve the problem Eq. 39-41:

$$R_{41} = K g_{31}, \; R_{51} = K g_{51}, \; R_{61} = K g_{61} \qquad (39)$$

$R_{32} = Kg_{32}, R_{72} = Kg_{72}, R_{82} = Kg_{82}$ (40)

$R_{41} = R_{61} = R_{72} = R_{82} = 0$ (41)

$K_1 = R_{51}g_{51}^{-1} \quad K_2 = R_{92}g_{91}^{-1}$

$$K_1 = \begin{bmatrix} 16.7204 & 17.5336 & -42.2947 & -41.2243 & -5.9932 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 157.8944 & 2.8791 & 20.9317 & 3.3627 & 5.7936 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Application to the induction machine: We propose to apply this approach to a real industrial system, an induction motor (5.5kW nominal output power, rated voltage (Y) 400V, rated current (Y): 11.20A, rated speed supported: 1445 rpm) controlled by a variable speed drive via a computer network. Delays prevent the establishment of command sequences:

Induction machine PARK model: The state representation according to the PARK model of the induction machine fed with an inverter is defined by the equation of state as follows: (Castillo-Toledoy *et al.*, 2004; Boucetta, 2008).

In this model:

$x \in R^p$ vector state; $u \in R^n$ control vector with

R_s = Stator resistance.
R_r = Rotor resistance.
L_s = Stator inductance.
L_r = rotor inductance.
M_{sr} = Mutual inductance.
ω_s = electrical synchronism pulsation.
C_i = Vector inverter switching command Eq. 42:

$$\begin{bmatrix} \dot{Isd} \\ \dot{Isq} \\ \dot{\varnothing rd} \\ \dot{\varnothing rd} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{-(RsLr^2 + RrMsr^2)}{Lr(LsLr-Msr^2)} & \omega s & \frac{RrMsr}{Lr(LsLr-Msr^2)} & \frac{Msr\omega}{(LsLr-Msr^2)} \\ -\omega s & \frac{-(RsLr^2+RrMsr^2)}{Lr(LsLr-Msr^2)} & -\omega\frac{Msr}{(LsLr-Msr^2)} & \frac{RrMsr}{Lr(LsLr-Msr^2)} \\ \frac{RrMsr}{Lr} & 0 & \frac{-Rr}{Lr} & \omega s-\omega \\ 0 & \frac{RrMsr}{Lr} & -(\omega s-\omega) & \frac{-Rr}{Lr} \end{bmatrix} \quad (42)$$

$$\begin{bmatrix} Isd \\ Isq \\ \varnothing rd \\ \varnothing rq \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{2}*LrE}{\sqrt{3}(LsLr-Msr^2)} & \frac{-\sqrt{2}LrE}{\sqrt{6}(LsLr-Msr^2)} & \frac{-\sqrt{2}LrE}{\sqrt{6}(LsLr-Msr^2)} \\ 0 & \frac{LrE}{\sqrt{6}(LsLr-Msr^2)} & \frac{-LrE}{\sqrt{6}(LsLr-Msr^2)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

Consequently, referring to previous models we can present a model of the induction machine voltage controlled we impose on the machine to follow a reference speed. The state variables chosen are the stator currents, The rotor flux and the mechanical angular pulsation: I_{sd}, I_{sq}, Φ_{rd}, Φ_{rq}, ω the system state is Eq. 43:

$$\begin{bmatrix} \dot{Isd} \\ \dot{Isq} \\ \dot{\varnothing rd} \\ \dot{\varnothing rd} \\ \dot{W} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{-(RsLr^2+RrMsr^2)}{Lr(LsLr-Msr^2)}Isd+\omega sIsq\frac{RrMsr}{Lr(LsLr-Msr^2)}\varnothing rd+\frac{Msrw}{(LsLr-Msr^2)}\varnothing rq \\ -\omega sIsd-\frac{(RsLr^2+RrMsr^2)}{Lr(LsLr-Msr^2)}Isq-\omega\frac{Msr}{(LsLr-Msr^2)}\varnothing rd+\frac{RrMsr}{Lr(LsLr-Msr^2)}\varnothing rq \\ \frac{RrMsr}{Lr}Isd-\frac{Rr}{Lr}\varnothing rd+(\omega s-\omega)\varnothing rq \\ \frac{RrMsr}{Lr}Isq-(\omega s-\omega)\varnothing rd-\frac{Rr}{Lr}\varnothing rq \\ \frac{P^2Msr}{LrJ}\varnothing rdIsq-\frac{P^2Msr}{LrJ}\varnothing rdIsq-\frac{P}{J}Cr-\frac{f}{J}\omega \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{2}*LrE}{\sqrt{3}(LsLr-Msr^2)} & \frac{-\sqrt{2}LrE}{\sqrt{6}(LsLr-Msr^2)} & \frac{-\sqrt{2}LrE}{\sqrt{6}(LsLr-Msr^2)} \\ 0 & \frac{LrE}{\sqrt{6}(LsLr-Msr^2)} & \frac{-LrE}{\sqrt{6}(LsLr-Msr^2)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (43)$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

The output vector, selected, is Eq 44:

$$y(t) = \begin{bmatrix} \varnothing rd \\ \varnothing rq \\ W \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} Isd \\ Isq \\ \varnothing rd \\ \varnothing rq \\ W \end{bmatrix} \quad (44)$$

The system is nonlinear because there are couplings between the pulsation ω and the flux Φ_{rq}, between ω and the flux Φ_{rd}, between the current I_{sq} and Φ_{rd} and another between I_{sd} and flux Φ_{rq}.

Linearization of the state model of the induction machine: Linearization of the state model of the machine comes from the Jacobian matrix. We determine the operating points and the linear state model around the nominal operating point. The system of linear state machine is Eq. 45:

$$\begin{cases} \dot{\tilde{x}} = A\tilde{x} + B\tilde{u} \\ \tilde{y} = C\tilde{x} \end{cases} \tag{45}$$

With:

$$A =$$

$$\begin{bmatrix}
\dfrac{-(RsLr^2 + RrMsr^2)}{Lr(LsLr - Msr^2)} & \omega s & \dfrac{RrMsr}{Lr(LsLr - Msr^2)} & \dfrac{Msr\bar{\omega}}{(LsLr - Msr^2)} & \dfrac{Msr\overline{\varnothing rq}}{(LsLr - Msr^2)} \\
-Ws & \dfrac{-(RsLr^2 + RrMsr^2)}{Lr(LsLr - Msr^2)} & -\bar{\omega}\dfrac{Msr}{(LsLr - Msr^2)} & \dfrac{RrMsr}{Lr(LsLr - Msr^2)} & \dfrac{Msr\overline{\varnothing rq}}{(LsLr - Msr^2)} \\
\dfrac{RrMsr}{Lr} & 0 & \dfrac{-Rr}{Lr} & \omega s - \bar{\omega} & -\overline{\varnothing rq} \\
0 & \dfrac{RrMsr}{Lr} & -(\omega s - \bar{\omega}) & \dfrac{-Rr}{Lr} & \overline{\varnothing rd} \\
\dfrac{-P^2Msr}{LrJ} & \dfrac{-P^2Msr}{LrJ}\overline{\varnothing rd} & \dfrac{-P^2Msr}{LrJ}\overline{Isq} & \dfrac{-P^2Msr}{LrJ}\overline{Isd} & \dfrac{-f}{j}
\end{bmatrix}$$

$$B = \begin{bmatrix}
\dfrac{\sqrt{2}*LrE}{\sqrt{3}(LsLr - Msr^2)} & \dfrac{-\sqrt{2}LrE}{\sqrt{6}(LsLr - Msr^2)} & \dfrac{-\sqrt{2}LrE}{\sqrt{6}(LsLr - Msr^2)} \\
0 & \dfrac{LrE}{\sqrt{6}(LsLr - Msr^2)} & \dfrac{-LrE}{\sqrt{6}(LsLr - Msr^2)} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\overline{Isq}, \overline{Isd}, \overline{\varnothing rq}, \overline{\varnothing rd}, \bar{\omega}$: The nominal operating values of the induction machine.

Control of the induction machine through a data transmission network : The objective is to study the stability of the system despite the occurrence of delays. The sampling period is defined in relation with the smallest constant time which is, $T_s = L_s / R_s$. and the switching period of the inverter given by Eq. 46:

$$T_c = 1/f_c \tag{46}$$

The system in discrete time is Eq. 47:

$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k) \\ y(k) = C_d x(k) + D_d u(k) \end{cases} \tag{47}$$

The delay is attributed to the network so it is introduced into the control u (k-τ). The system (47) becomes Eq. 48-50:

$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k-\tau) \\ y(k) = C_d x(k) + D_d u(k-\tau) \end{cases} \tag{48}$$

or $u(k-\tau) = -Kx(k-\tau)$ (49)

$$x(k+1) = A_d x(k) - B_d Kx(k-\tau) \tag{50}$$

By applying the method of switching system to find the control matrix K, Such the closed loop system for determining a delay is stable although, we use the equivalent condition to the condition of Lyapunov number (11).

For a delay $\tau = 1$ Eq. 51 and 52:

$$x(k+1) = A_d x(k) - B_d Kx(k-1) \tag{51}$$

$$\begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = \begin{bmatrix} A_d & -B_d K \\ I & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} \tag{52}$$

The matrices of system status switching written Eq. 53:

$$Z(k+1) = A_1 Z(k) \tag{53}$$

With $A_1 = \begin{bmatrix} A_d & -B_d k \\ I & 0 \end{bmatrix}$

We set $G1 = \begin{bmatrix} g_1 & g_2 \\ g_3 & g_4 \end{bmatrix}$ LMI condition for $\tau(k) = 1$ is Eq. 54:

$$\begin{bmatrix} G_1 + G_1^t - S & G_1^t A_1^t \\ A_1 G_1 & S \end{bmatrix}$$

$$\begin{bmatrix} G_1 + G_1^t - S & (*)^t \\ \begin{bmatrix} A_d g_1 - B_d K g_3 & A_d g_3 - B_d K g_4 \\ g_1 & g_1 \end{bmatrix} & S \end{bmatrix} > 0 \tag{54}$$

We set $g_3 = 0$ and $R_4 = Kg_4$ LMI condition becomes Eq. 55:

$$\begin{bmatrix} G_1 + G_1^t - S & G_1^t A_1^t \\ \begin{bmatrix} A_d g_1 & A_d g_3 - B_d K g_4 \\ g_1 & g_1 \end{bmatrix} & S \end{bmatrix} > 0 \tag{55}$$

The system is stable gain state feedback $K_1 = R_4 g_4^{-1}$ for a delay $\tau = 2$ Eq. 56-57:

$$x(k+1) = A_d x(k) - B_d K x(k-2) \tag{56}$$

$$\begin{bmatrix} x(k+1) \\ x(k) \\ x(k-1) \end{bmatrix} = \begin{bmatrix} A_d & 0 & -B_d K \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \\ x(k-2) \end{bmatrix} \tag{57}$$

The switched system is written Eq. 58:

$$Z(k+1) = A_2 Z(k) \tag{58}$$

With $A_2 = \begin{bmatrix} A_d & 0 & -B_d K \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$

The Lyapunov condition for $\tau = 2$ is Eq. 59:

$$\begin{bmatrix} G_2 + G_2^t - S & G_2^t + G_2^t \\ A_2 + G_2 & S \end{bmatrix} > 0 \tag{59}$$

We $G_2 = \begin{bmatrix} g_{12} & g_{22} & g_{32} \\ g_{42} & g_{52} & g_{62} \\ g_{72} & g_{82} & g_{92} \end{bmatrix}$ the condition for $\tau = 2$ are Eq. 60:

$$\begin{bmatrix} G_2 + G_2^t - S_2 & (*)^t \\ \begin{bmatrix} A_d g_{12} & A_d g_{22} & A_d g_{32} \\ -B_d K g_{72} & -B_d K g_{82} & -B_d K g_{92} \\ g_{12} & g_{22} & g_{32} \\ g_{42} & g_{52} & g_{61} \end{bmatrix} & S_2 \end{bmatrix} > 0 \tag{60}$$

The previous conditions inequalities are BMI, difficult to resolve. We propose the following conditions to come over this difficulty:

$$g_{72} = g_{82} = 0, \quad R_{92} = Kg_{92}$$

The system is stable for a gain: $K_2 = R_{92} g_{92}^{-1}$.

For a delay $\tau \in [0, 2]$, LMI conditions are Eq. 61-69:

$$\begin{bmatrix} G_2 + G_2^t - S_2 & (*)^t \\ \begin{bmatrix} A_d g_{12} & A_d g_{22} & A_d g_{32} \\ -B_d K g_{72} & -B_d K g_{82} & -B_d K g_{92} \\ g_{12} & g_{22} & g_{32} \\ g_{42} & g_{52} & g_{61} \end{bmatrix} & S_2 \end{bmatrix} > 0 \tag{61}$$

$$\begin{bmatrix} G_2 + G_2^t - S_2 & (*)^t \\ \begin{bmatrix} A_d g_{12} & A_d g_{22} & A_d g_{32} \\ -B_d K g_{72} & -B_d K g_{82} & -B_d K g_{92} \\ g_{12} & g_{22} & g_{32} \\ g_{42} & g_{52} & g_{61} \end{bmatrix} & S_1 \end{bmatrix} > 0 \tag{62}$$

$$\begin{bmatrix} G_2 + G_2^t - S_2 & (*)^t \\ \begin{bmatrix} A_d g_{12} & A_d g_{22} & A_d g_{32} \\ -B_d K g_{72} & -B_d K g_{82} & -B_d K g_{92} \\ g_{12} & g_{22} & g_{32} \\ g_{42} & g_{52} & g_{61} \end{bmatrix} & S_0 \end{bmatrix} > 0 \tag{63}$$

$$\begin{bmatrix} G_1 + G_1^t - S_1 & (*)^t \\ \begin{bmatrix} A_d g_{11} & A_d g_{21} & A_d g_{31} \\ -B_d K g_{41} & -B_d K g_{51} & -B_d K g_{61} \\ g_{11} & g_{21} & g_{31} \\ g_{41} & g_{51} & g_{61} \end{bmatrix} & S_1 \end{bmatrix} > 0 \qquad (64)$$

$$\begin{bmatrix} G_1 + G_1^t - S_1 & (*)^t \\ \begin{bmatrix} A_d g_{11} & A_d g_{21} & A_d g_{31} \\ -B_d K g_{41} & -B_d K g_{51} & -B_d K g_{61} \\ g_{11} & g_{21} & g_{31} \\ g_{41} & g_{51} & g_{61} \end{bmatrix} & S_2 \end{bmatrix} > 0 \qquad (65)$$

$$\begin{bmatrix} G_1 + G_1^t - S_1 & (*)^t \\ \begin{bmatrix} A_d g_{11} & A_d g_{21} & A_d g_{31} \\ -B_d K g_{41} & -B_d K g_{51} & -B_d K g_{61} \\ g_{11} & g_{21} & g_{31} \\ g_{41} & g_{51} & g_{61} \end{bmatrix} & S_0 \end{bmatrix} > 0 \qquad (66)$$

$$\begin{bmatrix} G_0 + G_0^t - S_0 & (*)^t \\ \begin{bmatrix} A_d g_{10} & A_d g_{20} & A_d g_{30} \\ -B_d K g_{10} & -B_d K g_{20} & -B_d K g_{30} \\ g_{10} & g_{20} & g_{30} \\ g_{40} & g_{50} & g_{60} \end{bmatrix} & S_0 \end{bmatrix} > 0 \qquad (67)$$

$$\begin{bmatrix} G_0 + G_0^t - S_0 & (*)^t \\ \begin{bmatrix} A_d g_{10} & A_d g_{20} & A_d g_{30} \\ -B_d K g_{10} & -B_d K g_{20} & -B_d K g_{30} \\ g_{10} & g_{20} & g_{30} \\ g_{40} & g_{50} & g_{60} \end{bmatrix} & S_1 \end{bmatrix} > 0 \qquad (68)$$

$$\begin{bmatrix} G_0 + G_0^t - S_0 & (*)^t \\ \begin{bmatrix} A_d g_{10} & A_d g_{20} & A_d g_{30} \\ -B_d K g_{10} & -B_d K g_{20} & -B_d K g_{30} \\ g_{10} & g_{20} & g_{30} \\ g_{40} & g_{50} & g_{60} \end{bmatrix} & S_2 \end{bmatrix} > 0 \qquad (69)$$

We verify Eq. 70:

$$G_2 = \begin{bmatrix} g_{12} & g_{22} & g_{32} \\ g_{42} & g_{52} & g_{62} \\ g_{72} & g_{82} & g_{92} \end{bmatrix};$$

$$G_1 = \begin{bmatrix} g_{11} & g_{21} & g_{31} \\ g_{41} & g_{51} & g_{61} \\ g_{71} & g_{81} & g_{91} \end{bmatrix}; G_0 = \begin{bmatrix} g_{10} & g_{20} & g_{30} \\ g_{40} & g_{50} & g_{60} \\ g_{70} & g_{80} & g_{90} \end{bmatrix} \qquad (70)$$

The previous conditions inequalities are BMI, difficult to resolve. We propose the following conditions:

$R_{10} = K g_{10}, R_{20} = K g_{20}, R_{30} = K g_{30}, R_{41} = K g_{41},$
$R_{51} = K g_{51}, R_{61} = K g_6, R_{32} = K g_{32}, R_{72} = K g_{72},$
$R_{82} = K g_{82}$
$R_{10} = R_{20} = R_{41} = R_{61} = R_{72} = R_{82} = 0$
$K_0 = R_{30} g_{30}^{-1} \quad K_1 = R_{51} g_{51}^{-1} \quad K_2 = R_{92} g_{92}^{-1}$

RESULTS AND DISCUSSION

Illustration: The study of the stability of model states of the induction machine with delays in the order, based on the use of the second Lyapunov method and the formulation of the problem of calculating the gain matrix inequality leads to not linear. Thus the approach of system for the model of states of the induction machine is used to calculate the feedback gain K for a delay that happens has five times the sampling period T_e. $T_c = 1/f_c = 100$ μs. ($F_c = 10$ KHz) To respect the Shannon theorem was, $T_e < 2 * T_c$ so we set $T_e = 100$ μs. The system in discrete time is that Eq. 71:

$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k) \\ y(k) = C_d x(k) + D_d u(k) \end{cases} \qquad (71)$$

With:

$$A_d = \begin{bmatrix} 0.9812 & 0.03091 & 0.0113 & 2.726 & 0.001175 \\ -0.03095 & 0.981 & -0.728 & 0.01308 & 0.0108 \\ 8.042e-05 & 1.512e-06 & 0.9994 & 0.001244 & -1.302e-05 \\ -1.123e-06 & 8.228e-05 & -000123 & 0.9994 & -0.0001195 \\ -0.002919 & -0.03137 & 0.3725 & 0.2837 & 0.9998 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 1.842 & -0.9853 & -0.9453 \\ -0.02891 & 1.608 & -1.579 \\ 7.5e-05 & -3.666e-05 & -3.829e-05 \\ -6.496e-07 & 6.625e-05 & -6.55e-05 \\ -0.00287 & -0.02387 & 0.02674 \end{bmatrix}$$

$C_d = C, D_d = D = 0$

By applying the method of switching system to find the control matrix K such that the closed loop system for a delay is stable although determining and using Matlab for solving LMI inequality, we find the following results:

For a delay τ = 1:

$$K_1 = R_4 g_4^{-1} = 1e - 05 *$$
$$\begin{bmatrix} 0.1913 & 0.6358 & 0.0704 & -0.3410 & 0.3105 \\ 0.1822 & 0.6079 & 0.0660 & -0.3280 & 0.2963 \\ 0.1824 & 0.6067 & 0.0668 & -0.3259 & 0.2961 \end{bmatrix}$$

For a delay τ = 2:

$$K_2 = R_9 g_9^{-1} = 1e - 04 *$$
$$\begin{bmatrix} 0.0716 & 0.1180 & 0.1069 & 0.0077 & 0.0521 \\ 0.0684 & 0.1127 & 0.1020 & 0.0073 & 0.0497 \\ 0.0683 & 0.1125 & 0.1020 & 0.0073 & 0.0497 \end{bmatrix}$$

For a delay τ = 3:

$$K_2 = R_{16} g_{16}^{-1} = 1e - 05 *$$
$$\begin{bmatrix} -0.1840 & -0.1790 & 0.3989 & 0.0615 & -0.2385 \\ -0.1755 & -0.1708 & 0.3807 & 0.0586 & -0.2277 \\ -0.1755 & -0.1707 & 0.3805 & 0.0587 & -0.2275 \end{bmatrix}$$

For a delay τ = 4:

$$K_4 = R_{25} g_{25}^{-1} = 1e - 04 *$$
$$\begin{bmatrix} -0.1101 & -0.1022 & -0.1772 & -0.1599 & -0.942 \\ -0.1052 & -0.0976 & -0.1692 & -0.1527 & -0.0899 \\ -0.1051 & -0.975 & -0.1690 & -0.1526 & -0.0898 \end{bmatrix}$$

For a delay τ = 5:

$$K_5 = R_{36} g_{36}^{-1} = 1.0e - 08 *$$
$$\begin{bmatrix} 0.0475 & 0.1102 & 0.0358 & -0.0133 & 0.0564 \\ 0.0453 & 0.1052 & 0.0342 & -0.0127 & 0.0538 \\ 0.0453 & 0.1051 & 0.0342 & -0.0127 & 0.0537 \end{bmatrix}$$

For a delay: τ ∈ [0, 2]:

$$K_0 = R_{30} g_{30}^{-1} = 1.0e - 004 *$$
$$\begin{bmatrix} -0.1365 & 0.0456 & 0.1375 & 0.6665 & 1.7229 \\ -0.1303 & 0.0436 & 0.1313 & 0.6364 & 1.6450 \\ -0.1302 & 0.0435 & 0.1312 & 0.6358 & 1.6435 \end{bmatrix}$$
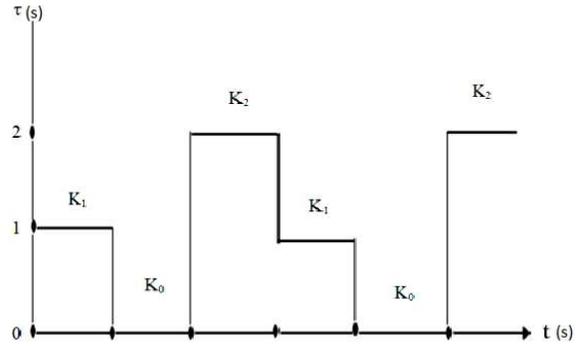

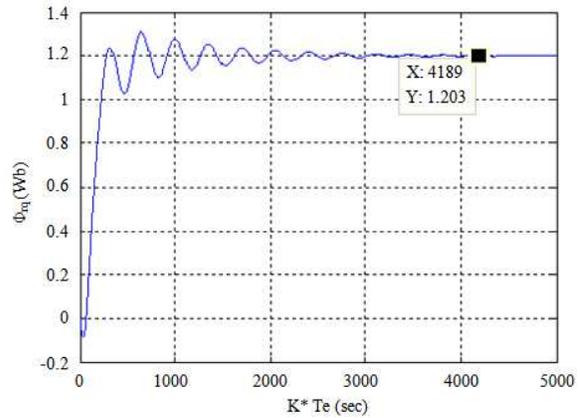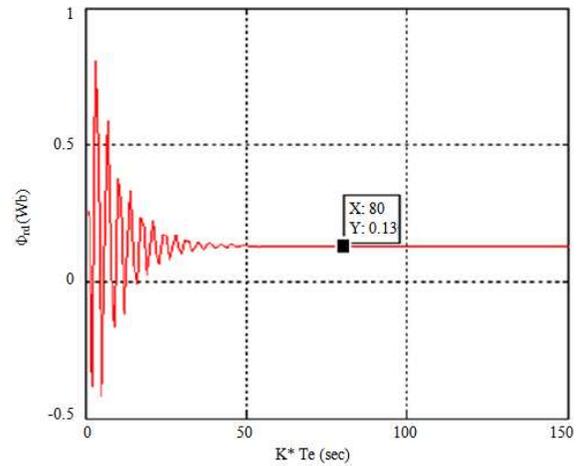Fig. 1: Switching gain K according to the delay

Fig. 2: Nominal Rotor flux along the axis "d'' and axis "q"

$$K_1 = R_{51} g_{51}^{-1} = 1.0e - 003 *$$
$$\begin{bmatrix} -0.3695 & 1.1142 & 0.2694 & 0.0574 & -1.2551 \\ -0.3528 & 1.0640 & 0.2573 & 0.0574 & -1.1985 \\ -0.3525 & 1.0629 & 0.2570 & 0.0548 & -1.1973 \end{bmatrix}$$
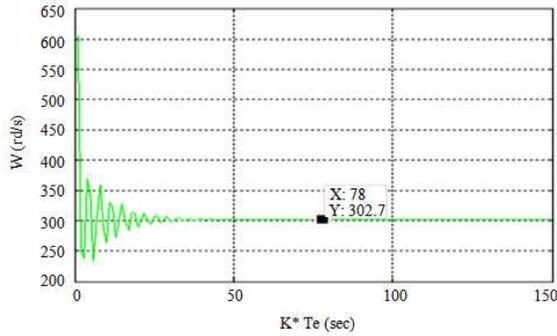
Fig. 3: Rated electric pulsation

$$K_2 = R_{92} g_{92}^{-1} =$$

$$\begin{bmatrix} 0.3945 & 0.4119 & 0.4171 & 0.3819 & 0.4143 \\ 0.3766 & 0.3933 & 0.3983 & 0.3646 & 0.3955 \\ 0.3763 & 0.3929 & 0.3979 & 0.3643 & 0.3952 \end{bmatrix}$$

Simulation and results: Matlab simulation of the system around the nominal operating point below for a delay $\tau \in [0, 2]$, (Fig. 1) gives the following signals:

The nominal operating point: $\begin{cases} Isd_n = 10.97A \\ Isq_n = 15.77A \\ \varnothing rd_n = 1.2Wb \\ \varnothing rq_n = 0.13Wb \\ W_n = 302.6rd/s \end{cases}$

According to the curves of the outputs Φ_{rq}, Φ_{rd} and ω system we find that the system is stable for a delay $\tau = 2$, (Fig. 2 and 3), the state feedback gain K_2 calculated by the method of switching system stabilizes the closed loop system for a delay of less than two sampling periods. This calculation method therefore offers a large range of stability because it tolerates delays in transmission networks $0 \leq \tau \leq 2$.

CONCLUSION

Remote control with stability problems is a new challenge for scientists. Indeed, despite the progress the case of nonlinear systems with remote-controlled continuous delays pose serious problems of stability and stabilization.

In this study, we proposed stabilization algorithms based on the use of hopping systems for the calculation of a gain stabilizing state feedback. The difficulty of this approach is to obtain non-linear matrix. Inequalities

require the contribution of relaxation that can reduce the field of possible solutions. The proposed method was tested on an induction machine supplied by a variable speed drive, despite the random delays introduced the system remains stable and retains the nominal performance.

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