American Journal of Engineering and Applied Sciences

Original Research Paper

Velocities and Accelerations at the 3R Mechatronic Systems

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Article history
Received: 02-01-2017
Revised: 09-01-2017
Accepted: 18-03-2017

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Abstract: This article presents an original method to determine the speeds and accelerations to structures MP R-3 The structure of the 3R (space) are known (required) rotation speeds of the triggers and must be determined speeds and accelerations of the endeffector M. Starting from the positions of direct kinematic system MP R-3deriving these system of relations in depending on the time, once and then a second time (the second derivation) is first obtains the speeds of the system and for the second time the accelerations endeffector point M. System on which must be resolved has three equations and three independent parameters to determine. Constructive basis is represented by a robot with three degrees of freedom (a robot with three axis of rotation). In the case where a study (analysis) a robot anthropomorphic with three axis of rotation (which represents the main movements, it is absolutely necessary), already has a system of the basis on which it can add other movements (secondary,). All calculations have been arranged and in the form of the array.

Keywords: Anthropomorphic Robots, Direct Kinematics, 3R Systems, Matrix Systems, Velocities, Accelerations

Introduction

The humanoids robots are used now as a tool for research in several scientific fields.

Researchers need to understand the structure of the human body and behavior (biomechanics) to build and to study robots humanoids. On the other hand the attempt simulation of the human body leads to a greater understanding of it. Human knowledge is a field of study, which is focused on the way in that people learn from sensory information in order to acquire the skills and insightful motor. Such knowledge is used to develop models for the calculation of human behavior and has been the improvement in time.

It has been suggested that robotics highly advanced will facilitate the increase in ordinary people.

With all that the original purpose of humanoid research has been to build a better orthosis and prosthesis for human beings, knowledge has been transferred between the two disciplines. Some examples are: Prosthesis footswitch with electrical adjustment for impaired neuromuscular, orthosis ankle-foot, biological realistic prosthesis leg and forearm prosthesis (Aversa *et al*., 2016a; 2016b; 2016c; 2016d; 2016e; 2016f).

In addition to the research, robots humanoids are developed to perform human activities, such as personal assistance, where they would be able to help places of work diseased and the elderly and dirty or dangerous. Work places ordinary, such as to be a yacht or a worker of a production line of cars are also suitable for the humanoids." In essence, as they can use tools and operate the equipment and vehicles designed to human form, those humanoids could carry out, theoretically, any load a human being may, as long as they have the software itself. However, the complexity to do this is deceptively big.

They are more popular for the provision of entertainment too. For example, Ursula, food Sex Female, sing, play music, dances and speaks to the public her from Universal Studios. More highlights Disney hire the use of animatrons, robots that look, move and speaking in the same way as human beings, in some thematic shows the parking brake.

These animatrons look so realistic that it can be difficult to decipher the remote whether or not they are in fact they are human. Though they look realistic, they do not have any cognitive autonomy or natural. Various robots humanoids and possible their applications in everyday life are presented in a documentary film independently, called Plug and Pray, which has been launched in 2010.

Robots humanoids, in particular with the algorithms of artificial intelligence, could be useful for future dangerous mission and/or at a distance of the spatial scan without the need to turn around again and to get back on the ground once the mission is completed.

A sensor is a device which measures some attribute of the world. As one of the three primitives of robotics (apart from the planning and control), detection plays an important role in the fault finding sequential paradigms.

Sensors can be classified on the basis of the physical process which works with or, depending on the type of metering information which they give that output. In this case, was used by the second approach.

Proprioceptive sensors sense the position, the orientation and speed of the body and the rubber humanoid him.

Human beings otoliths channels and the semi-circular (in his ear internal) are used to maintain balance and orientation. In addition people do not use their own proprioceptive sensors (e.g., to the touch, muscular extension, limb position) to help with robots Humanoid orientation. Their uses accelerometers to measure the acceleration, from which the speed can be calculated by means of the integration; tilt sensors to measure the tilt sensor; of force placed in her arms and legs to measure the force of contact with the robot environment; position sensors, which indicates the actual robot position (from which the speed can be calculated by the derivation) or even the speed sensors.

The arrays tactels can be used to provide data on what has been reached. The shadow of the Hand uses an array of 34 tactile arranged under the skin of polyurethane on each finger. Touch sensors also provide information about the forces and the torques transferred between the robot and the other objects.

The vision refers to the processing of data from any way that uses the electromagnetic spectrum to produce an image. In the robots humanoids is used to recognize the objects and determine their properties (The sensors to the works at more than in a similar way with the eyes of human beings). Most robots humanoids use CCD cameras that the sensors.

Sensors allow sound robots humanoids to hear the speech and the sounds of the environment and to carry out that the ears of the human being. Microphones are usually used for this task.

Actuators are the motors responsible for the movement in the robot.

Robots humanoids are constructed in such a way that they mimics the human body, so that they can use the actuators which carried out such as the muscles and joints, though with a different structure. To obtain the same effect as the human movement, robots humanoids use actuator in rotating main. They may be either electrical wiring, pneumatic, hydraulics, piezoelectric ultrasound or.

Actuators hydraulic and electrical have a behavior very rigid and may be made only to act in a manner consistent with the, through the use of strategies relatively complex for the control of the feedback. While the electrical components of the motor actuation using cored are more suited for high speed and low load, hydraulic works well at low speed and high load.

Elements of the piezoelectric actuator generate a movement with a large capacity of force, when it is applied to the voltage. They can be used for positioning the ultra-fine and for generating and handling large forces or pressure in situations static and dynamic.

Elements of the actuator with ultrasound are designed to produce movements in an order micrometer at frequencies ultrasound (over 20 kHz). They are useful for vibration control applications, positioning and fast switching.

Elements of the pneumatic actuator operate based on the compressibility of gas. As they are inflated, extend along the axis and how to deflate, contracts. In the case where an end is fixed, the other will move in a linear trajectory. These components are intended for low speed and low load/average. Between the components of the pneumatic actuator are: Cylinders the gaiter, motors pneumatic, stepper motors gauge and of the artificial muscles pneumatic.

In the planning and control, the essential difference between the humanoids and other types of robots (such as industrial), is the fact that the robot move must be human consumption as it may be, using locomotion with feet, in particular lever biped. Planning the ideal for the movements of the humanoids during the normal course should lead to a minimal power consumption, as it happens in the human body. For this reason, the studies on the dynamics and control of these types of structures are becoming increasingly important.

The problem of walking the stabilization scythe robots on the surface is of great importance. Maintenance of the center of gravity of the robot over the center of the camp in order to ensure a stable position can be chosen as an objective of the control.

In order to maintain the dynamic balance during their walk and a robot needs information on the contact force and the movement to the actual and desired. The solution to this problem is based on a major concept, Zero Point Time (ZMP).

Another feature of the robots humanoids is that moves, gather information (using sensors) on "real world and to interact with her. They do not remain as other manipulators robots who works in environments very structured. In order to enable the humanoids to move in complex environments, planning and control must focus on the detection of self-collision, planning and the way of avoiding obstacles.

The humanoids have not yet been some features of the human body. These include structures with variable flexibility to provide a fuse (to the robot in itself and for the people) and redundancy movements, i.e., more degrees of freedom and availability task, therefore, at the level. With all that these features are desirable for the robots humanoids, they will bring more complexity and new problems of planning and control. The field of dealing with the control of the whole body with these problems and to address proper coordination of many degrees of freedom, for example in order to carry out more tasks simultaneously control, while in the following an order given of priority.

Robotic screwing unit with automatic feeding of screws, are automatic machines with anthropomorphic arms: Extremely flexible in all aspects. They allow to screw on different planes and have a high reconversion factor: In case of change of product or mode of production, the arm can be used in the most diverse applications (Fig. 1).

Anthropomorphic industrial robots have become the most prevalent and most used.

They are most prevalent across the planet because they were very well put in place and are more easily designed, manufactured and implemented, compared to other types of robots and manipulators.

The most common is the structure of Fig. 1, with a base made up of three rotating elements, 3R.

It is a mechanical structure, furniture, with three degrees of mobility, easy designed, with a high mobility and a large work space.

They are big advantages, it has established itself in the world of industrial robots and was generalized.

Like all industrial robots and this anthropomorphic structure, it was launched on the auto industry, which commissioned and produced almost all modern industrial robots.

The main advantages of a structure of this kind are great mobility, a wider working space, a good dynamic, fast moving and acceptable accuracy for industrial operations daily conjunction with most common.

When it comes to reliability and stability excessive anthropomorphic structure can't cope, she successfully being replaced by parallel structures.

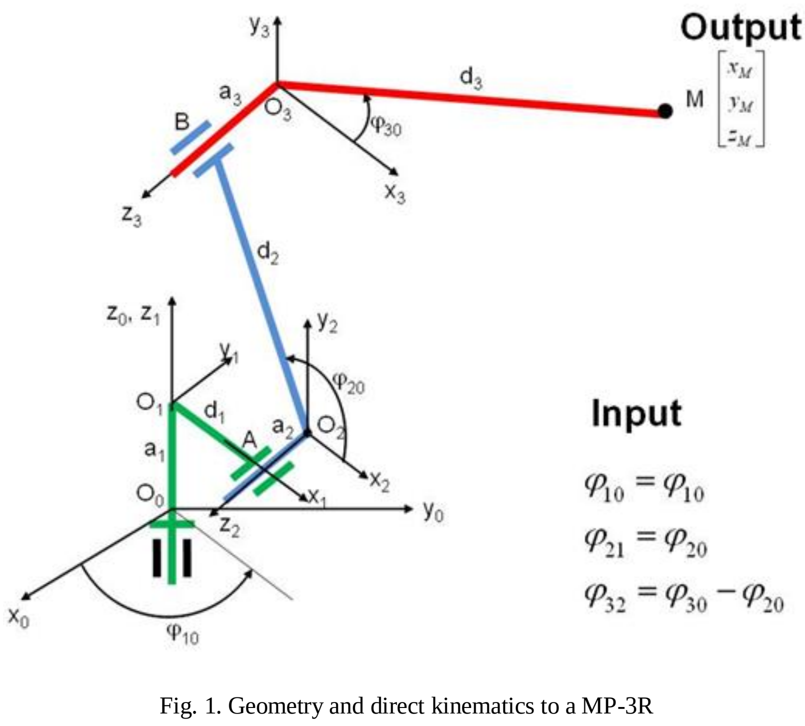

Fig. 1. Geometry and direct kinematics to a MP-3R

Even if parallel or mixed platforms are more accurate and more stable than an anthropomorphic structure, yet they can't be used everywhere, is more expensive, more difficult designed, built and implemented.

However anthropomorphic mechanical structures have a high mobility, good dynamics and a low complexity, which has imposed in industrial robotic almost everyone, except for some special cases, the necessary structures are parallel or mixed ones.

In this study we present the basis of an anthropomorphic structures, in terms of construction, geometric and kinematic.

With all the robots as anthropomorphic, have different forms of structural in recent years have been developed especially those with movements of rotation, three or more axis. Constructive basis is represented by a robot with three degrees of freedom (a robot with three axis of rotation) (Antonescu, 2000).

In the case where a study (analysis) a robot anthropomorphic with three axis of rotation (which represents the main movements, absolutely necessary), we already have a system of the basis on which we can then add other movements (secondary,).

The basic system has three axis of rotation: A vertical axis (via this axis and your whole system is turned, for positioning) and two horizontal axis (each making it possible to a rotation of the arm). The calculations have been arranged and in the form of the array.

In the direct kinematics picture known cinematic parameters (input parameters), which are absolute angles of rotation of the three moving components: φ_{10}, φ_{20}, φ_{30}, the angles of rotation of the three elements of the actuator (electric motors, mounted in kinematic couplings of rotation) and parameters determine (output parameters) are the coordinates of the three absolutely x_M, y_M, z_M from the point M, i.e., cinematic parameters (coordinated) of the endeffector (which can be a hand, to get, a pinch of bonding, painted, cut, etc).

In the reverse cinematic (Angeles *et al*., 1989; Borrel and Liegeois, 1986; Do and Yang, 1988; Guglielmetti and Longchamp, 1994; Hollerbach, 1983; Petrescu *et al*., 2009; Seeger, 1990) already knows the coordinates of the x_M, y_M, z_M from the point M and must be determined independent rotations of the three moving parts φ_{10}, φ_{20}, φ_{30}, on the basis of parameters cinematic imposed endeffector x_M, y_M, z_M, known (forced).

Having angles determined independent, is then to calculate the relative movements of rotation, of the three motors to drive, the rotating shift (Petrescu *et al*., 2009).

Having regard to the positions of the already established, it is appropriate to the problem of the determination of the gear selection and accelerations of the system (Petrescu and Petrescu, 2015).

Determining the Positions at the 3R Robots (Mechatronic Systems)

The movement of manipulators the serial number and the robots will be illustrated by a model 3R drive train (Fig. 1), a system of medium difficulty, ideal for the understanding of the phenomenon, but also to specify the basic skills necessary to begin the calculations for systems simpler and more complex.

The coordinate system has been noted with $x_0O_0y_0z_0$. Mobile systems concerned (strengthened with) the three moving components (1, 2, 3) have indices 1, 2 and 3. Their orientation was chosen as convenient way. The parameters of the known cinematic (input parameters in the direct kinematic) are absolute angles of rotation of the three moving components: φ_{10}, φ_{20}, φ_{30}, the angles of rotation of the three elements of the actuator (electric motors, mounted in kinematic couplings of rotation). The parameters determine (output parameters) are the coordinates of the three absolutely x_M, y_M, z_M from the point M, i.e., cinematic parameters (coordinated) of the endeffector (which can be a hand, to get, a pinch of bonding, painted, cut, etc).

To begin to write the array of vectors (A_{01}), which changes the coordinates of the origin of the coordinate system by moving the linear (travel) from O_0 to O_1, where the axs remain parallel between them standing committee (Equation 1):

$$A_{01} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \tag{1}$$

One writes T_{01} the array of rotation, which rotates in the $x_1O_1y_1z_1$ in relation to the $x_0O_0y_0z_0$ (this is a square matrix of 3×3, see the relationship 2):

$$T_{01} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} \cos\phi_{10} & -\sin\phi_{10} & 0 \\ \sin\phi_{10} & \cos\phi_{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

On the first column (which represents the coordinates of the rotated axis O_1x_1) it writes the coordinates of the unit vector of O_1x_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O_1 but without rotation; see the relationship 3):

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \quad (3)$$

On the second column of the matrix T_{01} it writes the coordinates of the unit vector of the rotated axis O_1y_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O_1 but without rotation system; see the relationship 4):

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} \quad (4)$$

On the third column of the matrix T_{01} it writes the coordinates of the unit vector of the rotated axis O_1z_1 in rapport of the old system $x_0O_0y_0z_0$ (translated into O_1 but without rotation system; see the relationship 5):

$$\begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \end{bmatrix} \quad (5)$$

In the elected case (Fig. 1), the unit vector of the rotated axis O_1x_1, has in rapport of the old system $x_0O_0y_0z_0$, translated into O_1 without rotation, the coordinates given by the column unit vector (relationship 6):

$$\begin{bmatrix} \alpha_x = 1\cdot\cos\phi_{10} = \cos\phi_{10} \\ \alpha_y = 1\cdot\sin\phi_{10} = \sin\phi_{10} \\ \alpha_z = 1\cdot\cos 90^0 = 1\cdot 0 = 0 \end{bmatrix} \quad (6)$$

The unit vector of the rotated axis O_1y_1, has in rapport of the old system of axes $x_0O_0y_0z_0$ (translated into O1 without rotation), coordinates data unit vector column (relationship 7):

$$\begin{bmatrix} \beta_x = 1\cdot\cos(\pi/2+\phi_{10}) = -\sin\phi_{10} \\ \beta_y = 1\cdot\sin(\pi/2+\phi_{10}) = \cos\phi_{10} \\ \beta_z = 1\cdot\cos(\pi/2) = 1\cdot 0 = 0 \end{bmatrix} \quad (7)$$

The unit vector of the rotated axis O_1z_1 has in rapport of the old system of axes $x_0O_0y_0z_0$ (translated into O_1 without rotation), coordinates data unit vector column (relationship 8):

$$\begin{bmatrix} \gamma_x = 1\cdot\cos 90^0 = 1\cdot 0 = 0 \\ \gamma_y = 1\cdot\cos 90^0 = 1\cdot 0 = 0 \\ \gamma_z = 1\cdot\cos 0^0 = 1\cdot 1 = 1 \end{bmatrix} \quad (8)$$

See the obtained matrix T_{01} (relationship 2).

Transition from the coordinate system $x_1O_1y_1z_1$ to the coordinate system $x_2O_2y_2z_2$ is done in two distinct phases. The first phase is a translation of the entire system so that (axes being parallel with them itself) the center O_1 to move into the center O_2; then the second stage in which it done the rotation of system of axes and the center O remains fixed permanently.

The translation of the system from point 1 to the point 2 (see the relationship 9) is doing by the column vector, matrix A_{12}:

$$A_{12} = \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} \quad (9)$$

On the old O_1x_1 axis O_2 has been moved with d1, on the old axis O_1y_1 O_2 has been moved with a_2 and on the old $_{O1}z_1$ axis O_2 has not been moved.

The unit vector of the O_2x_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated but not rotated) the next coordinates (expression 10):

$$\alpha_x = 1; \quad \alpha_y = 0; \quad \alpha_z = 0 \quad (10)$$

The unit vector of the O_2y_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated in O_2 but not rotated) the next coordinates (expression 11):

$$\beta_x = 0; \quad \beta_y = 0; \quad \beta_z = 1 \quad (11)$$

The unit vector of the O_2z_2 axis has in rapport of the old system $x_1O_1y_1z_1$ (translated in O_2 but not rotated) the coordinates given by the expression 12:

$$\gamma_x = 0; \quad \gamma_y = -1; \quad \gamma_z = 0 \quad (12)$$

The transfer square matrix (the rotation matrix: T12) is writing with relationship 13:

$$T_{12} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad (13)$$

Transition from the coordinate system $x_2O_2y_2z_2$ to the coordinate system $x_3O_3y_3z_3$ is done in two distinct phases. The first phase is a translation of the entire system so that (axes being parallel with them itself) the center O_2 to move into the center O_3; then the second stage in which it done the rotation of system of axes and the center O_3 remains fixed permanently.

First O_2 is moving into O_3 (axes being parallel with them itself; see the relationship 14):

$$A_{23} = \begin{bmatrix} d_2 \cdot \cos\phi_{20} \\ d_2 \cdot \sin\phi_{20} \\ -a_3 \end{bmatrix} \tag{14}$$

Then O_3 remains fixed and the axes of coordinate system are rotating. The unit vector of the O_3x_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the α coordinates (see expression 15):

$$\alpha_x = 1; \quad \alpha_y = 0; \quad \alpha_z = 0 \tag{15}$$

The unit vector of the O_3y_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the β coordinates (see relationship 16):

$$\beta_x = 0; \quad \beta_y = 1; \quad \beta_z = 0 \tag{16}$$

The unit vector of the O_3z_3 axis has in rapport of the coordinate system $x_2O_2y_2z_2$ (translated in O_3 but not rotated) the γ coordinates (see relationship 17):

$$\gamma_x = 0; \quad \gamma_y = 0; \quad \gamma_z = 1 \tag{17}$$

In the model from the Fig. 1 the system $x_3O_3y_3z_3$ has not been rotated in rapport of the system $x_2O_2y_2z_2$ (from 2 to 3 held just a translation). In this case the rotation matrix is the unit matrix (expression 18):

$$T_{23} = \begin{bmatrix} \alpha_x & \beta_x & \gamma_x \\ \alpha_y & \beta_y & \gamma_y \\ \alpha_z & \beta_z & \gamma_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

The column vector matrix that positions the point M in the coordinate system $x_3O_3y_3z_3$ is written with relation 19:

$$X_{3M} = \begin{bmatrix} x_{3M} \\ y_{3M} \\ z_{3M} \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \tag{19}$$

Coordinates of the point M in the system (2) $x_2O_2y_2z_2$ are obtained by a transformation matrix which is having the form (20):

$$X_{2M} = A_{23} + T_{23} \cdot X_{3M} \tag{20}$$

First, is performed the matrix product (relations 21):

$$T_{23} \cdot X_{3M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \tag{21}$$

Then, will be calculated X_{2M} (relationship 22):

$$\begin{aligned} X_{2M} &= A_{23} + T_{23} \cdot X_{3M} \\ &= \begin{bmatrix} d_2 \cdot \cos\phi_{20} \\ d_2 \cdot \sin\phi_{20} \\ -a_3 \end{bmatrix} + \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \\ -a_3 \end{bmatrix} \end{aligned} \tag{22}$$

Coordinates of the point M in the system (1) $x_1O_1y_1z_1$ are obtained by the relationships (23-25):

$$X_{1M} = A_{12} + T_{12} \cdot X_{2M} \tag{23}$$

$$\begin{aligned} T_{12} \cdot X_{2M} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \\ -a_3 \end{bmatrix} \\ &= \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ a_3 \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \end{aligned} \tag{24}$$

$$\begin{aligned} X_{1M} &= A_{12} + T_{12} \cdot X_{2M} \\ &= \begin{bmatrix} d_1 \\ a_2 \\ 0 \end{bmatrix} + \begin{bmatrix} d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ a_3 \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \\ &= \begin{bmatrix} d_1 + d_2 \cdot \cos\phi_{20} + d_3 \cdot \cos\phi_{30} \\ a_2 + a_3 \\ d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix} \end{aligned} \tag{25}$$

Coordinates of the point M in the fixed system $x_0O_0y_0z_0$, are written with the relationships (26-27, 28-29):

$$X_{0M} = A_{01} + T_{01} \cdot X_{1M} \tag{26}$$

$$T_{01}\cdot X_{1M}=\begin{bmatrix}\cos\phi_{10} & -\sin\phi_{10} & 0\\ \sin\phi_{10} & \cos\phi_{10} & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}d_1+d_2\cdot\cos\phi_{20}+d_3\cdot\cos\phi_{30}\\ a_2+a_3\\ d_2\cdot\sin\phi_{20}+d_3\cdot\sin\phi_{30}\end{bmatrix}\tag{27}$$

$$T_{01}\cdot X_{1M}=\begin{bmatrix}(d_1+d_2\cdot\cos\phi_{20}+d_3\cdot\cos\phi_{30})\cdot\cos\phi_{10}-(a_2+a_3)\cdot\sin\phi_{10}\\ (d_1+d_2\cdot\cos\phi_{20}+d_3\cdot\cos\phi_{30})\cdot\sin\phi_{10}+(a_2+a_3)\cdot\cos\phi_{10}\\ d_2\cdot\sin\phi_{20}+d_3\cdot\sin\phi_{30}\end{bmatrix}\tag{28}$$

$$X_{0M}=A_{01}+T_{01}\cdot X_{1M}=\begin{bmatrix}0\\0\\a_1\end{bmatrix}+\begin{bmatrix}(d_1+d_2\cdot\cos\phi_{20}+d_3\cdot\cos\phi_{30})\cdot\cos\phi_{10}-(a_2+a_3)\cdot\sin\phi_{10}\\ (d_1+d_2\cdot\cos\phi_{20}+d_3\cdot\cos\phi_{30})\cdot\sin\phi_{10}+(a_2+a_3)\cdot\cos\phi_{10}\\ d_2\cdot\sin\phi_{20}+d_3\cdot\sin\phi_{30}\end{bmatrix}=\begin{bmatrix}(d_1+d_2\cdot\cos\phi_{20}+d_3\cdot\cos\phi_{30})\cdot\cos\phi_{10}-(a_2+a_3)\cdot\sin\phi_{10}\\ (d_1+d_2\cdot\cos\phi_{20}+d_3\cdot\cos\phi_{30})\cdot\sin\phi_{10}+(a_2+a_3)\cdot\cos\phi_{10}\\ a_1+d_2\cdot\sin\phi_{20}+d_3\cdot\sin\phi_{30}\end{bmatrix}\tag{29}$$

X_{0M} is arranged in the form (30):

$$X_{0M}=\begin{bmatrix}x_M\\y_M\\z_M\end{bmatrix}\begin{bmatrix}d_1\cdot\cos\phi_{10}-a_2\cdot\sin\phi_{10}+d_2\cdot\cos\phi_{20}\cdot\cos\phi_{10}\\ -a_3\cdot\sin\phi_{10}+d_3\cdot\cos\phi_{30}\cdot\cos\phi_{10}\\ d_1\cdot\sin\phi_{10}+a_2\cdot\cos\phi_{10}+d_2\cdot\cos\phi_{20}\cdot\sin\phi_{10}\\ +a_3\cdot\cos\phi_{10}+d_3\cdot\cos\phi_{30}\cdot\sin\phi_{10}\\ a_1+d_2\cdot\sin\phi_{20}+d_3\cdot\sin\phi_{30}\end{bmatrix}\tag{30}$$

The same calculations will be presented now by a direct method (having in view the matrix calculations 31):

$$\begin{aligned}X_{0M}&=A_{01}+T_{01}\cdot X_{1M}=A_{01}+T_{01}\cdot(A_{12}+T_{12}\cdot X_{2M})\\&=A_{01}+T_{01}\cdot A_{12}+T_{01}\cdot T_{12}\cdot X_{2M}\\&=A_{01}+T_{01}\cdot A_{12}+T_{01}\cdot T_{12}\cdot(A_{23}+T_{23}\cdot X_{3M})\\&=A_{01}+T_{01}\cdot A_{12}+T_{01}\cdot T_{12}\cdot A_{23}+T_{01}\cdot T_{12}\cdot T_{23}\cdot X_{3M}\end{aligned}\tag{31}$$

It keeps the relationship (32):

$$\begin{aligned}X_{0M}&=A_{01}+T_{01}\cdot A_{12}\\&+T_{01}\cdot T_{12}\cdot A_{23}+T_{01}\cdot T_{12}\cdot T_{23}\cdot X_{3M}\end{aligned}\tag{32}$$

Now, one performs the matrix multiplications from expression 32 (relationships 33-37):

$$T_{01}\cdot A_{12}=\begin{bmatrix}\cos\phi_{10} & -\sin\phi_{10} & 0\\ \sin\phi_{10} & \cos\phi_{10} & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}d_1\\a_2\\0\end{bmatrix}=\begin{bmatrix}d_1\cdot\cos\phi_{10}-a_2\cdot\sin\phi_{10}\\ d_1\cdot\sin\phi_{10}+a_2\cdot\cos\phi_{10}\\ 0\end{bmatrix}\tag{33}$$

$$T_{01}\cdot T_{12}=\begin{bmatrix}\cos\phi_{10} & -\sin\phi_{10} & 0\\ \sin\phi_{10} & \cos\phi_{10} & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&0&-1\\0&1&0\end{bmatrix}=\begin{bmatrix}\cos\phi_{10} & 0 & \sin\phi_{10}\\ \sin\phi_{10} & 0 & -\cos\phi_{10}\\ 0 & 1 & 0\end{bmatrix}\tag{34}$$

$$T_{01}\cdot T_{12}\cdot A_{23}=\begin{bmatrix}\cos\phi_{10} & 0 & \sin\phi_{10}\\ \sin\phi_{10} & 0 & -\cos\phi_{10}\\ 0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}d_2\cdot\cos\phi_{20}\\ d_2\cdot\sin\phi_{20}\\ -a_3\end{bmatrix}=\begin{bmatrix}d_2\cdot\cos\phi_{10}\cdot\cos\phi_{20}-a_3\cdot\sin\phi_{10}\\ d_2\cdot\sin\phi_{10}\cdot\cos\phi_{20}+a_3\cdot\cos\phi_{10}\\ d_2\cdot\sin\phi_{20}\end{bmatrix}\tag{35}$$

$$T_{01}\cdot T_{12}\cdot T_{23}=\begin{bmatrix}\cos\phi_{10} & 0 & \sin\phi_{10}\\ \sin\phi_{10} & 0 & -\cos\phi_{10}\\ 0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\cos\phi_{10} & 0 & \sin\phi_{10}\\ \sin\phi_{10} & 0 & -\cos\phi_{10}\\ 0 & 1 & 0\end{bmatrix}\tag{36}$$

$$T_{01}\cdot T_{12}\cdot T_{23}\cdot X_{3M}=\begin{bmatrix}\cos\phi_{10} & 0 & \sin\phi_{10}\\ \sin\phi_{10} & 0 & -\cos\phi_{10}\\ 0 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}d_3\cdot\cos\phi_{30}\\ d_3\cdot\sin\phi_{30}\\ 0\end{bmatrix}=\begin{bmatrix}d_3\cdot\cos\phi_{10}\cdot\cos\phi_{30}\\ d_3\cdot\sin\phi_{10}\cdot\cos\phi_{30}\\ d_3\cdot\sin\phi_{30}\end{bmatrix}\tag{37}$$

The expression (32) takes the form (38):

$$X_{0M} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} + \begin{bmatrix} d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} \\ d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} \\ 0 \end{bmatrix}$$
$$+ \begin{bmatrix} d_2 \cdot \cos\phi_{10} \cdot \cos\phi_{20} - a_3 \cdot \sin\phi_{10} \\ d_2 \cdot \sin\phi_{10} \cdot \cos\phi_{20} + a_3 \cdot \cos\phi_{10} \\ d_2 \cdot \sin\phi_{20} \end{bmatrix}$$
$$+ \begin{bmatrix} d_3 \cdot \cos\phi_{10} \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{10} \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \end{bmatrix} = \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} \quad (38)$$
$$= \begin{bmatrix} d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \cos\phi_{10} \\ -a_3 \cdot \sin\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \cos\phi_{10} \\ d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} + d_2 \cdot \cos\phi_{20} \cdot \sin\phi_{10} \\ +a_3 \cdot \cos\phi_{10} + d_3 \cdot \cos\phi_{30} \cdot \sin\phi_{10} \\ a_1 + d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{bmatrix}$$

By the direct kinematics is obtained Cartesian coordinates x_M, y_M, z_M of the point M (the endeffector) in rapport with the three independent angular displacements φ_{10}, φ_{20}, φ_{30}, obtained using actuators (relationships 39-40):

$$\begin{cases} x_M = f_x(\phi_{10}, \ \phi_{20}, \ \phi_{30}) \\ y_M = f_y(\phi_{10}, \ \phi_{20}, \ \phi_{30}) \\ z_M = f_z(\phi_{10}, \ \phi_{20}, \ \phi_{30}) \end{cases} \quad (39)$$

$$\begin{cases} x_M = d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} \\ +d_2 \cdot \cos\phi_{20} \cdot \cos\phi_{10} - a_3 \cdot \sin\phi_{10} \\ +d_3 \cdot \cos\phi_{30} \cdot \cos\phi_{10} \\ \\ y_M = d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} \\ +d_2 \cdot \cos\phi_{20} \cdot \sin\phi_{10} + a_3 \cdot \cos\phi_{10} \\ +d_3 \cdot \cos\phi_{30} \cdot \sin\phi_{10} \\ \\ z_M = a_1 + d_2 \cdot \sin\phi_{20} + d_3 \cdot \sin\phi_{30} \end{cases} \quad (40)$$

Calculations are performed with absolute angular movements (φ_{10}, φ_{20}, φ_{30}), but the actuators movements do not match (all) with the independent angular movements. They are determined as follows (expressions 41):

$$\begin{cases} \phi_{10} = \phi_{10} \\ \phi_{21} = \phi_{20} \\ \phi_{32} = \phi_{30} - \phi_{20} \end{cases} \quad (41)$$

The first two actuators relative rotations coincide with the independent rotations (used in calculations), but the third actuator relative rotation is obtained as a difference between two absolute rotations (expressions 41). The velocities and the accelerations are obtained by the derivatives of the positions expressions (40) in rapport of the time.

Determining the Velocities and Accelerations at the 3R Robots

It starts from the relationship matrix gear (42) already known:

$$\begin{aligned} X_{0M} &= A_{01} + T_{01} \cdot X_{1M} = A_{01} + T_{01} \cdot (A_{12} + T_{12} \cdot X_{2M}) \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot X_{2M} \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot (A_{23} + T_{23} \cdot X_{3M}) \\ &= A_{01} + T_{01} \cdot A_{12} + T_{01} \cdot T_{12} \cdot A_{23} + T_{01} \cdot T_{12} \cdot T_{23} \cdot X_{3M} \end{aligned} \quad (42)$$

This is written as (43) simplified:

$$X_{0M} = A_{01} + P_1 + P_2 + T_{03} \cdot X_{3M} \quad (43)$$

Where:

$$A_{01} = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \quad (44)$$

$$P_1 = \begin{bmatrix} d_1 \cdot \cos\phi_{10} - a_2 \cdot \sin\phi_{10} \\ d_1 \cdot \sin\phi_{10} + a_2 \cdot \cos\phi_{10} \\ 0 \end{bmatrix} \quad (45)$$

$$P_2 = \begin{bmatrix} d_2 \cdot \cos\phi_{10} \cdot \cos\phi_{20} - a_3 \cdot \sin\phi_{10} \\ d_2 \cdot \sin\phi_{10} \cdot \cos\phi_{20} + a_3 \cdot \cos\phi_{10} \\ d_2 \cdot \sin\phi_{20} \end{bmatrix} \quad (46)$$

$$T_{03} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \quad (47)$$

$$X_{3M} = \begin{bmatrix} x_{3M} \\ y_{3M} \\ z_{3M} \end{bmatrix} = \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \quad (48)$$

The Velocities

It derives the relationship (39) a matrix and obtain expression (49):

$$\begin{aligned} \dot{X}_{0M} &= \dot{A}_{01} + \dot{P}_1 + \dot{P}_2 + \dot{T}_{03} \cdot X_{3M} + T_{03} \cdot \dot{X}_{3M} \\ &= \dot{P}_1 + \dot{P}_2 + \dot{T}_{03} \cdot X_{3M} + T_{03} \cdot \dot{X}_{3M} \\ &= \dot{P}_{12} + \dot{T}_{03} \cdot X_{3M} + T_{03} \cdot \dot{X}_{3M} \end{aligned} \quad (49)$$

Seeing that:

$$\dot{A}_{01} = \begin{bmatrix} 0 \\ 0 \\ \dot{a}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \tag{50}$$

$$\dot{P}_1 = \begin{bmatrix} -d_1 \cdot \sin\phi_{10} \cdot \omega_{10} - a_2 \cdot \cos\phi_{10} \cdot \omega_{10} \\ d_1 \cdot \cos\phi_{10} \cdot \omega_{10} - a_2 \cdot \sin\phi_{10} \cdot \omega_{10} \\ 0 \end{bmatrix} \tag{51}$$

$$\dot{P}_2 = \begin{bmatrix} -d_2 \cdot \sin\phi_{10} \cdot \omega_{10} \cdot \cos\phi_{20} \\ -d_2 \cdot \cos\phi_{10} \cdot \sin\phi_{20} \cdot \omega_{20} - a_3 \cdot \cos\phi_{10} \cdot \omega_{10} \\ d_2 \cdot \cos\phi_{10} \cdot \omega_{10} \cdot \cos\phi_{20} \\ -d_2 \cdot \sin\phi_{10} \cdot \sin\phi_{20} \cdot \omega_{20} - a_3 \cdot \sin\phi_{10} \cdot \omega_{10} \\ d_2 \cdot \cos\phi_{20} \cdot \omega_{20} \end{bmatrix} \tag{52}$$

$$\dot{T}_{03} = \begin{bmatrix} -\sin\phi_{10} \cdot \omega_{10} & 0 & \cos\phi_{10} \cdot \omega_{10} \\ \cos\phi_{10} \cdot \omega_{10} & 0 & \sin\phi_{10} \cdot \omega_{10} \\ 0 & 0 & 0 \end{bmatrix} \tag{53}$$

$$\dot{X}_{3M} = \begin{bmatrix} \dot{x}_{3M} \\ \dot{y}_{3M} \\ \dot{z}_{3M} \end{bmatrix} = \begin{bmatrix} -d_3 \cdot \sin\phi_{30} \cdot \omega_{30} \\ d_3 \cdot \cos\phi_{30} \cdot \omega_{30} \\ 0 \end{bmatrix} \tag{54}$$

$$\dot{P}_{12} = \dot{P}_1 + \dot{P}_2 = \begin{bmatrix} -d_1 \sin\phi_{10}\omega_{10} - a_2 \cos\phi_{10}\omega_{10} - a_3 \cos\phi_{10}\omega_{10} \\ -d_2 \sin\phi_{10}\omega_{10} \cos\phi_{20} - d_2 \cos\phi_{10} \sin\phi_{20}\omega_{20} \\ d_1 \cos\phi_{10}\omega_{10} - a_2 \sin\phi_{10}\omega_{10} - a_3 \sin\phi_{10}\omega_{10} \\ +d_2 \cos\phi_{10}\omega_{10} \cos\phi_{20} - d_2 \sin\phi_{10} \sin\phi_{20}\omega_{20} \\ d_2 \cos\phi_{20}\omega_{20} \end{bmatrix} \tag{55}$$

The following two products is determined: Matrix (56 and 57), from equation (49):

$$\dot{T}_{03} \cdot X_{3M} = \begin{bmatrix} -\sin\phi_{10} \cdot \omega_{10} & 0 & \cos\phi_{10} \cdot \omega_{10} \\ \cos\phi_{10} \cdot \omega_{10} & 0 & \sin\phi_{10} \cdot \omega_{10} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} = \begin{bmatrix} -d_3 \cdot \sin\phi_{10} \cdot \omega_{10} \cdot \cos\phi_{30} \\ d_3 \cdot \cos\phi_{10} \cdot \omega_{10} \cdot \cos\phi_{30} \\ 0 \end{bmatrix} \tag{56}$$

$$T_{03} \cdot \dot{X}_{3M} = \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -d_3 \cdot \sin\phi_{30} \cdot \omega_{30} \\ d_3 \cdot \cos\phi_{30} \cdot \omega_{30} \\ 0 \end{bmatrix} = \begin{bmatrix} -d_3 \cdot \cos\phi_{10} \cdot \sin\phi_{30} \cdot \omega_{30} \\ -d_3 \cdot \sin\phi_{10} \cdot \sin\phi_{30} \cdot \omega_{30} \\ d_3 \cdot \cos\phi_{30} \cdot \omega_{30} \end{bmatrix} \tag{57}$$

One can determine now $\dot{X}_{0M}$ (system 58):

$$\dot{X}_{0M} = \begin{bmatrix} (-d_1 \sin\phi_{10}\omega_{10} - a_2 \cos\phi_{10}\omega_{10} - a_3 \cos\phi_{10}\omega_{10} \\ -d_2 \sin\phi_{10}\omega_{10} \cos\phi_{20} - \\ -d_2 \cos\phi_{10} \sin\phi_{20}\omega_{20} - d_3 \sin\phi_{10}\omega_{10} \cos\phi_{30} \\ -d_3 \cos\phi_{10} \sin\phi_{30}\omega_{30}) \\ \\ (d_1 \cos\phi_{10}\omega_{10} - a_2 \sin\phi_{10}\omega_{10} - a_3 \sin\phi_{10}\omega_{10} \\ +d_2 \cos\phi_{10}\omega_{10} \cos\phi_{20} - \\ -d_2 \sin\phi_{10} \sin\phi_{20}\omega_{20} + d_3 \cos\phi_{10}\omega_{10} \cos\phi_{30} \\ -d_3 \sin\phi_{10} \sin\phi_{30}\omega_{30}) \\ \\ (d_2 \cos\phi_{20}\omega_{20} + d_3 \cos\phi_{30}\omega_{30}) \end{bmatrix} \tag{58}$$

The Accelerations

Follow relations accelerations. It derives the relation (49) to give the expression (59):

$$\begin{aligned} \ddot{X}_{0M} &= \ddot{P}_{12} + \ddot{T}_{03} \cdot X_{3M} + \dot{T}_{03} \cdot \dot{X}_{3M} \\ &+ \dot{T}_{03} \cdot \dot{X}_{3M} + T_{03} \cdot \ddot{X}_{3M} \\ &= \ddot{P}_{12} + \ddot{T}_{03} \cdot X_{3M} + 2 \cdot \dot{T}_{03} \cdot \dot{X}_{3M} + T_{03} \cdot \ddot{X}_{3M} \end{aligned} \tag{59}$$

$$\ddot{P}_{12} = \ddot{P}_1 + \ddot{P}_2 = \begin{bmatrix} (-d_1 \cos\phi_{10}\omega_{10}^2 + a_2 \sin\phi_{10}\omega_{10}^2 + a_3 \sin\phi_{10}\omega_{10}^2 \\ -d_2 \cos\phi_{10}\omega_{10}^2 \cos\phi_{20} + +d_2 \sin\phi_{10}\omega_{10} \sin\phi_{20}\omega_{20} \\ +d_2 \sin\phi_{10}\omega_{10} \sin\phi_{20}\omega_{20} - d_2 \cos\phi_{10} \cos\phi_{20}\omega_{20}^2) \\ \\ (-d_1 \sin\phi_{10}\omega_{10}^2 - a_2 \cos\phi_{10}\omega_{10}^2 - a_3 \cos\phi_{10}\omega_{10}^2 \\ -d_2 \sin\phi_{10}\omega_{10}^2 \cos\phi_{20} - d_2 \cos\phi_{10}\omega_{10} \sin\phi_{20}\omega_{20} \\ -d_2 \cos\phi_{10}\omega_{10} \sin\phi_{20}\omega_{20} - d_2 \sin\phi_{10} \cos\phi_{20}\omega_{20}^2) \\ \\ (-d_2 \sin\phi_{20}\omega_{20}^2) \end{bmatrix} \tag{60}$$

$$\ddot{T}_{03} = \begin{bmatrix} -\cos\phi_{10} \cdot \omega_{10}^2 & 0 & -\sin\phi_{10} \cdot \omega_{10}^2 \\ -\sin\phi_{10} \cdot \omega_{10}^2 & 0 & \cos\phi_{10} \cdot \omega_{10}^2 \\ 0 & 0 & 0 \end{bmatrix} \tag{61}$$

$$\ddot{X}_{3M} = \begin{bmatrix} -d_3 \cdot \cos\phi_{30} \cdot \omega_{30}^2 \\ -d_3 \cdot \sin\phi_{30} \cdot \omega_{30}^2 \\ 0 \end{bmatrix} \tag{62}$$

$$2 \cdot \dot{T}_{03} \cdot \dot{X}_{3M} = \begin{bmatrix} 2 \cdot d_3 \cdot \sin\phi_{10} \cdot \omega_{10} \cdot \sin\phi_{30} \cdot \omega_{30} \\ -2 \cdot d_3 \cdot \cos\phi_{10} \cdot \omega_{10} \cdot \sin\phi_{30} \cdot \omega_{30} \\ 0 \end{bmatrix} \tag{63}$$

$$\begin{aligned} &\ddot{T}_{03} \cdot X_{3M} \\ &= \begin{bmatrix} -\cos\phi_{10} \cdot \omega_{10}^2 & 0 & -\sin\phi_{10} \cdot \omega_{10}^2 \\ -\sin\phi_{10} \cdot \omega_{10}^2 & 0 & \cos\phi_{10} \cdot \omega_{10}^2 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} d_3 \cdot \cos\phi_{30} \\ d_3 \cdot \sin\phi_{30} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -d_3 \cdot \cos\phi_{10} \cdot \omega_{10}^2 \cdot \cos\phi_{30} \\ -d_3 \cdot \sin\phi_{10} \cdot \omega_{10}^2 \cdot \cos\phi_{30} \\ 0 \end{bmatrix} \end{aligned} \tag{64}$$

$$\begin{aligned} T_{03} \cdot \ddot{X}_{3M} &= \begin{bmatrix} \cos\phi_{10} & 0 & \sin\phi_{10} \\ \sin\phi_{10} & 0 & -\cos\phi_{10} \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -d_3 \cdot \cos\phi_{30} \cdot \omega_{30}^2 \\ -d_3 \cdot \sin\phi_{30} \cdot \omega_{30}^2 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -d_3 \cdot \cos\phi_{10} \cdot \cos\phi_{30} \cdot \omega_{30}^2 \\ -d_3 \cdot \sin\phi_{10} \cdot \cos\phi_{30} \cdot \omega_{30}^2 \\ -d_3 \cdot \sin\phi_{30} \cdot \omega_{30}^2 \end{bmatrix} \end{aligned} \tag{65}$$

It obtains the matrix of the endeffector accelerations (66) in function of the three actuators rotations (angular positions and velocities).

With $\omega_{10} = ct,\ \omega_{20} = ct,\ \omega_{30} = ct$:

$$\ddot{X}_{0M} = \begin{bmatrix} (-d_1 \cos\phi_{10}\omega_{10}^2 + a_2 \sin\phi_{10}\omega_{10}^2 + a_3 \sin\phi_{10}\omega_{10}^2 \\ -d_2 \cos\phi_{10}\omega_{10}^2 \cos\phi_{20} + 2d_2 \sin\phi_{10}\omega_{10} \sin\phi_{20}\omega_{20} \\ -d_2 \cos\phi_{10} \cos\phi_{20}\omega_{20}^2 + 2d_3 \sin\phi_{10}\omega_{10} \sin\phi_{30}\omega_{30} \\ -d_3 \cos\phi_{10}\omega_{10}^2 \cos\phi_{30} - d_3 \cos\phi_{10} \cos\phi_{30}\omega_{30}^2) \\ (-d_1 \sin\phi_{10}\omega_{10}^2 - a_2 \cos\phi_{10}\omega_{10}^2 - a_3 \cos\phi_{10}\omega_{10}^2 \\ -d_2 \sin\phi_{10}\omega_{10}^2 \cos\phi_{20} - 2d_2 \cos\phi_{10}\omega_{10} \sin\phi_{20}\omega_{20} \\ -d_2 \sin\phi_{10} \cos\phi_{20}\omega_{20}^2 - 2d_3 \cos\phi_{10}\omega_{10} \sin\phi_{30}\omega_{30} \\ -d_3 \sin\phi_{10}\omega_{10}^2 \cos\phi_{30} - d_3 \sin\phi_{10} \cos\phi_{30}\omega_{30}^2) \\ (-d_2 \sin\phi_{20}\omega_{20}^2 - d_3 \sin\phi_{30}\omega_{30}^2) \end{bmatrix} \tag{66}$$

Discussion

Anthropomorphic industrial robots have become the most prevalent and most used.

They are most prevalent across the planet because they were very well put in place and are more easily designed, manufactured and implemented, compared to other types of robots and manipulators.

The most common is the structure of Fig. 1, with a base made up of three rotating elements, 3R.

It is a mechanical structure, furniture, with three degrees of mobility, easy designed, with a high mobility and a large work space.

They are big advantages, they have established theyself in the world of industrial robots and was generalized.

Like all industrial robots and this anthropomorphic structure, it was launched on the auto industry, which commissioned and produced almost all modern industrial robots.

The main advantages of a structure of this kind are great mobility, a wider working space, a good dynamic, fast moving and acceptable accuracy for industrial operations daily conjunction with most common.

When it comes to reliability and stability excessive anthropomorphic structure can't cope, she successfully being replaced by parallel structures.

Even if parallel or mixed platforms are more accurate and more stable than an anthropomorphic structure, yet they can't be used everywhere, is more expensive, more difficult designed, built and implemented.

However anthropomorphic mechanical structures have a high mobility, good dynamics and a low complexity, which has imposed in industrial robotic almost everyone, except for some special cases, the necessary structures are parallel or mixed ones.

In this study has been presented the basis of an anthropomorphic structure, in terms of construction, geometric and kinematic.

This article presents an original method to determine the speeds and accelerations to structures MP R-3. The structure of the 3R (space) are known (required) rotation speeds of the triggers and must be determined speeds and accelerations of the endeffector M.

Starting from the positions of direct kinematic system MP R-3deriving these systems of relations in depending on the time, once and then a second time (the second derivation) is first obtains the speeds of the system and for the second time the accelerations endeffector point M.

System on which must be resolved has three equations and three independent parameters to determine. Constructive basis is represented by a robot with three degrees of freedom (a robot with three axis of rotation).

In the case where a study (analysis) a robot anthropomorphic with three axis of rotation (which represents the main movements, it is absolutely necessary), already has a system of the basis on which it can add other movements (secondary,). All calculations have been arranged and in the form of the array.

Conclusion

Constructive basis is represented by a robot with three degrees of freedom (a robot with three axis of rotation).

In the case where a study (analysis) a robot anthropomorphic with three axis of rotation (which represents the main movements, it is absolutely necessary), already has a system of the basis on which it can add other movements (secondary,).

All calculations have been arranged and in the form of the array.

Kinematics of the anthropomorphic systems with velocities and accelerations, may be solved by a basic model 3R, which is a spatial model with matrix calculations (which were presented on this work), or on a 2R planar, simplified model (Petrescu and Petrescu, 2015).

Acknowledgment

This text was acknowledged and appreciated by Assoc. Pro. Taher M. Abu-Lebdeh, North Carolina A and T State Univesity, United States, Samuel P. Kozaitis, Professor and Department Head at Electrical and Computer Engineering, Florida Institute of Technology, United States, whom we thank and in this way.

Funding Information

Research contract: Contract number 36-5-4D/1986 from 24IV1985, beneficiary CNST RO (Romanian National Center for Science and Technology) Improving dynamic mechanisms.

Contract research integration. 19-91-3 from 29.03.1991; Beneficiary: MIS; TOPIC: Research on designing mechanisms with bars, cams and gears, with application in industrial robots.

Contract research. GR 69/10.05.2007: NURC in 2762; theme 8: Dynamic analysis of mechanisms and manipulators with bars and gears.

Labor contract, no. 35/22.01.2013, the UPB, "Stand for reading performance parameters of kinematics and dynamic mechanisms, using inductive and incremental encoders, to a Mitsubishi Mechatronic System" "PN-II-IN-CI-2012-1-0389".

Author's Contributions

All the authors contributed equally to prepare, develop and carry out this manuscript.

Ethics

This article is original. Authors declare that are not ethical issues that may arise after the publication of this manuscript.

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