

Original Research Paper

# Solving the Robust Design Problem for a Two-Commodity Flow Network with Node Failure

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**Abstract:** The robust design in a flow network is one of the most important problems. It is defined as searching the optimal capacity that can be assigned to the nodes such that the network still survived even under the node's failure. This problem is considered NP-hard. So, this study presents a genetic-based algorithm to determine the maximum node capacity for a two-commodity flow network with node failure. I.e., searching the minimum sum of the assigned capacities and the maximum network reliability. The obtained results show that The proposed GA-based algorithm succeeded to solve the robust problem for the two-commodity flow network considering the node's failure.

**Keywords:** Flow Network, Robust Design, Two-Commodity, Node Failure, Genetic Algorithm

## Introduction

Reliability evaluation is important to arrange the scheduling in many network systems (Zuo *et al.*, 2007). In the case of an existing flow between nodes, the system reliability of a flow network is defined as the probability that the maximum flow path throw (nodes or arcs) is not less than the given demand ( $d$ ) (Younes and Hassan, 2011; Lin *et al.*, 1995; Lin, 2001). Lin (2001) presented a simple algorithm for generating all lower boundary points to calculate system reliability. Lin (2007a) constructed a stochastic-flow network to model the information system. Lin (2007b) proposed a new performance index, the probability that the upper bound of the system capacity equals the demand vector subject to the budget constraint, to evaluate the quality level of a multicommodity limited-flow network. Satishan and Kapur, (2006) proposed an algorithm to search for lower boundary points and used it to account for the exact reliability.

Lin (2002) Presented an algorithm to evaluate the reliability in terms of minimal paths for a two-commodity ( $d_1, d_2$ ) by generating all lower boundary points for a stochastic-flow network in which each arc and node has several capacities and may fail. Hassan and Abdou, (2020) presented a method to calculate the maximum value of the demand ( $d_{max}$ ). The simple algorithm used to find ( $d_{max}$ ), its idea is based on cut sets.

In (Chen and Lin, 2008) the research debated the capacity assignment problem and offered a definition to the critical node. Prove that if the critical node fails then

the system reliability drops to zero. In general, the Capacity Assignment Problem (CAP) is defined as the search for the optimal capacities that minimize the lateness in the network and maximize its reliability (Zantuti, 2008). Study structural analysis is very important to determine the critical nodes to avoid the system reliability don't drop to zero (Chen and Lin, 2010). Structural analysis is used to identify the node's structural and locate critical nodes to avoid the system reliability don't drop to zero (Chen and Lin, 2010).

The robust design is defined as an issue in quality engineering where the designed product functions well even in a versatile environment. However, in the context of a stochastic-flow network, it means the network functions well even in a node's failure situation (Chen and Lin, 2010). The robustness of an assignment is therefore recognized as the fact that the critical nodes generated after the assignment are equal to the important structural nodes (Chen and Lin, 2010; Niu *et al.*, 2020). The Robust Design Problem (RDP) in a capacitated flow network is to search for the minimum capacity assigned to each edge such that the network still survived even under the edge's failure (Chen, 2012).

Genetic Algorithms (GAs) are an engaging tool to solve optimization problems (Back and Schwefel, 1996). The genetic algorithm will use a search technique to find optimized or tuned or approximate solutions and Genetic algorithms are an important technique in searching for the optimum option from a set of solutions available for a particular design, (Gen and Chen, 2000; Gong *et al.*, 2016; Azaron *et al.*, 2009).

Younes and Hassan, (2011) present a GA to compute the reliability of a stochastic-flow network in which each arc or node has several capacities and may fail. To reach the best results in reliability optimization, to solve reliability optimization problems, many based GA approaches were proposed such as (Chen, 2012; Deeter and Smith, 1997; Dengiz *et al.*, 1997). Also, GA is used to solve multiple objective optimization problems (Altıparmak *et al.*, 1998; Coello and Christiansen, 2000). In addition, GA is used to solve the capacity assignment problem and the objective is to minimize the total capacities while meeting the network reliability requirement (Hamed *et al.*, 2020).

In this study, we extend the robust design problem to a flow network with multiple demands, (Lin, 2009). Also, we propose an approach based on GA to solve the presented problem. The main objective of the proposed GA is to search for the best-assigned capacities such that the network reliability is maximized. I.e., maximize the reliability and minimize the total assigned capacities.

We used the GAs for their characteristics that distinguish them from the rest of the algorithms. GA is based on the concept of natural selection and natural genetics, these are easy to understand and enforcement, these are widely used in optimization problems, these work on a population of points rather than an individual point, these use probabilistic transition rule instead of deterministic rules, GAs can potent deal with a large number of variables. These are more effective for complex problems than simple problems (Sharma, 2013).

The paper is organized as follows: We proposed notations and assumptions in section 2. Section 3 presents the problem description. In section 4 we clarify the problem formulation. Section 5 describes the GA components. The proposed GA algorithm is given in section 6. Studied cases illustrate in section 7. Finally, we present the conclusion in section 8.

## Notations and Assumptions

### Notations

$np$	No. of paths
$mp_j$	Minimal paths no. $j, j = 1, 2, \dots, np$
$w_i^k$	The weight of commodity ( $k = 1, 2$ )
$d_1$	The demand of commodity 1
$d_2$	The demand of commodity 2
$d$	The demand, $d = ((w_i^1 * d_1) + (w_i^2 * d_2))$
$neq$	No. of components (nodes)
$n$	The node
$F1$	Flow vector; $F1 = (f_1^1, f_2^1, \dots, f_{np}^1)$
$F2$	Flow vector; $F2 = (f_1^2, f_2^2, \dots, f_{np}^2)$
$X$	Capacity vector; $X = (x_1, x_2, \dots, x_{neq})$
$r$	The probability

$R_{d_1, d_2}$	System reliability to the given demands; $d_1, d_2$
$NP$	Number of chromosomes
$NG$	Number of genes equals to $neq$
$maxgene$	The maximum number of generations
$cr$	The GA crossover rate
$mr$	The GA mutation rate

### Assumptions

The following assumptions should be satisfied for the given SFN:

- Two types of commodity are transmitted from source to sink
- Capacities of different components are statistically independent
- The capacity of each node is an integer
- Flow of each commodity must satisfy the flow-conservation law (Fulkerson and Ford, 1962)

## Problem Description

### The Reliability Evaluation to Two Commodities

Given the demand  $d_1, d_2$ , the system reliability  $R_{d_1, d_2}$  is defined by, (Lin, 2002):

$$R_{d_1, d_2} = \Pr\{X | V(X) \geq (d_1, d_2)\} \quad (1)$$

where,  $X$  is a lower boundary point for  $d_1, d_2$ .

And  $X$  can be deduced from  $F1 = (f_1^1, f_2^1, \dots, f_{np}^1)$  and  $F2 = (f_1^2, f_2^2, \dots, f_{np}^2)$  by using the following equation:

$$x_i = \left\{ \left[ \sum_{j=1}^{np} (w_j^1 f_j^1 + w_j^2 f_j^2) \right] | n_i \in mp_j \right\} \quad (2)$$

for each  $i = 1, 2, \dots, neq$

when  $F$  generate by equations that satisfies the flowing two constraints:

$$\left\{ \left[ \sum_{j=1}^{np} (w_j^1 f_j^1 + w_j^2 f_j^2) \right] | n_i \in mp_j \right\} \leq M^1 \quad (3)$$

for  $i = 1, 2, \dots, neq$

$$\sum_{j=1}^{np} f_j^k = d_k \quad (k = 1, 2) \quad (4)$$

### The Capacity Assignment

The network structure is important to assignment the capacity. To analyze the network structure, we determine the minimum paths. The minimum path is an ordered sequence of nodes without a cycle. From the paths we specify the node covering, which extends to the definition of Structural Impact Factor (SIF)  $S_i$  for node  $n_i$ .

### Coverage Node

When  $n_i, n_j \in N$  and  $n_j \subseteq n_i$ ,  $n_j$  is said to be covered by  $n_i$  if no flow passes through  $n_j$  when no flow passes through  $n_i$ , (Chen and Lin, 2010; Niu *et al.*, 2020; Chen, 2012).

### Critical Node

When the capacity node  $n_i \leq d$  and  $R_{d_1, d_2, i}$  is zero then this node is critical node (Chen and Lin, 2010; Chen, 2012).

### Probabilities Calculation Planner for Each Node

At first, we must be defining the probability for  $\Pr\{x_i\}$  of node  $n_i$  to calculate  $R_{d_1, d_2, i}$ . To produce the corresponding capacities, we suppose find  $c_i$  components for nodes  $n_i$ . Each component has the reliability of  $r_i$ , (Chen and Lin, 2010). Then the probability for the current capacity  $x_i$  is  $\Pr\{x_i\}$  and is denoted as:

$$\Pr\{x_i = k\} = \binom{c_i}{k} r_i^k (1 - r_i)^{c_i - k} \quad (5)$$

### Problem Formulation

Let  $M = (M^1, M^2, \dots, M^{neq})$  as assigned capacities to the set of nodes  $(n_1, n_2, \dots, n_{neq})$ . The mathematical formulation of the problem is:

$$\text{Minimize } \mathbb{C} \quad (6)$$

$$\text{s.t.} \quad (7)$$

$$R_{d_1, d_2} \text{ is maximized}$$

where,  $\mathbb{C} = \sum_i^n M^i$  and  $R_{d_1, d_2}$  is reliability corresponding to the assigned capacities under demand  $d = (w_1^1 d_1 + w_2^2 d_2)$ .  $M^i$  ranges from 1 to  $d$ , except for critical nodes (Chen and Lin, 2008),  $M^i = d$ .

### The GA Components

The following subsections depict different components of the proposed GA.

#### Representation

A chromosome representation is needed to describe each chromosome in the GA algorithm. The chromosome  $M$  is exemplifying by a series of length  $(neq)$ , where  $(neq)$  is the number of nodes as shown in this equation  $(M^1, M^2, \dots, M^{neq})$ .

#### Initial Population

The generation of an initial population is the first step in a GA. Each chromosome in generation encodes a possible solution to a problem. After creating the

initial population, each chromosome is evaluated and assigned a fitness value according to the fitness function, (Diaz-Gomez and Hougen, 2007).

#### Fitness Function

We will use the total sum of capacities  $\mathbb{C}$  for each chromosome  $i$  as a fitness function, i.e.,  $fit(i) = \mathbb{C}$ . For the selection process, the fitness function is normalized as follows:

Calculate the sum of fitness for all chromosomes in the population,  $sum\_fit$ .

```
For i = 1 to NP
    normalize_fit(i) = fit(i)/sum_fit
End for
```

#### Selection

Selection process picks the fittest solutions by giving it a high percentage when picking the next generation and thus leads to choosing the best solution to the problem. In this research, we will use roulette wheel selection to choose two parents based on cumulative sum (*cumsum*) as follows:

```
Begin
For i = 1 to NP
    For j = i to NP
        cumsum(i) = cumsum(i) + normalize_fit(j)
    End for j
End for i
Generate a random number  $\mu \in [0,1]$ 
For i = 1 to NP
    If  $\mu > cumsum(i)$ 
        Parent1 = i-1.
    End if
End for i
Generate a random number  $\mu \in [0,1]$ 
For i = 1 to NP
    If  $\mu > cumsum(i)$ 
        Parent2 = i-1.
    End if
End for i
End
```

#### Crossover

Crossover process (Lim *et al.*, 2017) is vital in generating new chromosomes by selecting two parents called parent 1 and parent 2. During crossover the parent chromosomes are taken in pairs and exchange information between two parents and generate offspring 1 and offspring 2. These offspring become next generation parent chromosomes, (Varun Kumar and Panneerselvam, 2017).

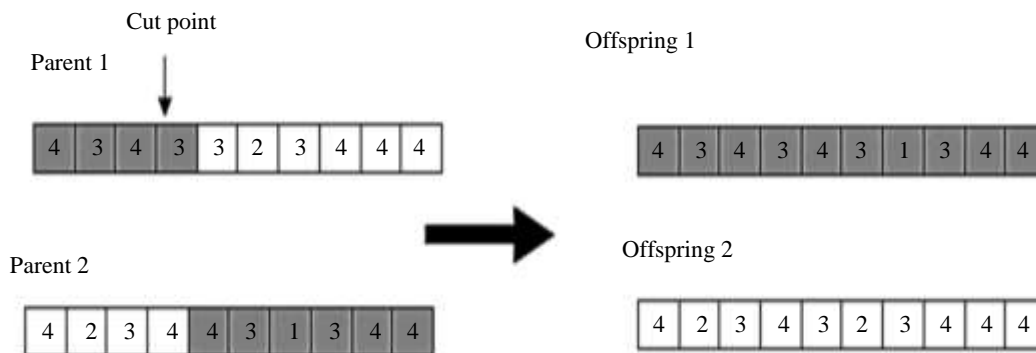


Fig. 1: Crossover process

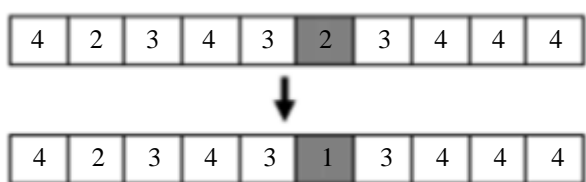


Fig. 2: Mutation process

In this research, we will use single point crossover, a crossover point is randomly generated which determines the point for swap of information between two parents to generate two offspring as shown in Fig. 1.

### Mutation

Mutation takes place after crossover is done. Mutation used to modify the genes of a chromosome selected with a mutation probability  $mr$ . It takes place the changes randomly to one “genes” to generate a new offspring. In this research, we will use Creep Mutation as shown in Fig. 2. In creep mutation, the gene is selected random and the value of it is changed with a random value between  $[1: d]$  unless the value of it, (Soni and Kumar, 2014).

## The Proposed GA Algorithm

Start

Input  $d_1, d_2, w_i^k, neq, MPs, r_i,$

and components' informations.

then compute the demand  $d$

Determine the critical nodes in the network.

Set the GA parameters:  $NP, NG, maxgene, cr$  and  $mr$ .

Generate the initial population randomly.

Evaluate the initial population.

While  $g \leq maxgene$ , do

While  $p \leq NP$ , do

Select two chromosomes.

Generate new offspring after applying crossover and mutation.

```

    p = p + 1
  end do
  Evaluate the current population.
  Save the best solutions.
  g = g + 1
End do
Report the best solutions found.
End
    
```

## Studied Cases

In the following subsections, the proposed GA has been applied to four networks. We studied and observed the results in the case of varying each GA parameter. We found that the best values for these parameters are:  $Maxgene = 30, NP = 10, cr = 0.95$  and  $mr = 0.05$ .

### Four-Node Network

This network has four nodes as shown in Fig. 3. The network has four minimal paths as follows:  $mp_1 = \{n_1, n_2, n_3, n_4\}, mp_2 = \{n_1, n_2, n_3, n_4\}, mp_3 = \{n_1, n_2, n_3, n_4\}, mp_4 = \{n_1, n_2, n_3, n_4\}$ . The nodes probability,  $r_i = (0.99, 0.98, 0.98, 0.99), w_i^l = 1, w_i^l = 1.5$ , Table 1 shows the best candidate M with the corresponding  $\mathbb{C}, R_{3,2}$ , execution time, Table 2 shows the best candidate M with the corresponding  $\mathbb{C}, R_{3,2}$ , execution time.

### Eight-Node Network

This network has eight nodes as shown in Fig. 4. The network has eight minimal paths as follows:  $mp_1 = \{n_1, n_2, n_4, n_8\}, mp_2 = \{n_1, n_2, n_5, n_8\}, mp_3 = \{n_1, n_2, n_6, n_8\}, mp_4 = \{n_1, n_2, n_7, n_8\}, mp_5 = \{n_1, n_3, n_4, n_8\}, mp_6 = \{n_1, n_3, n_5, n_8\}, mp_7 = \{n_1, n_2, n_6, n_8\}, mp_8 = \{n_1, n_2, n_7, n_8\}$ . The nodes' probabilities,  $r_i = (0.90, 0.88, 0.93, 0.91, 0.89, 0.90, 0.87, 0.92), w_i^l = 1, w_i^l = 1.5$ . Table 3 shows the best candidate M with the corresponding  $\mathbb{C}, R_{1,2}$ , execution time.

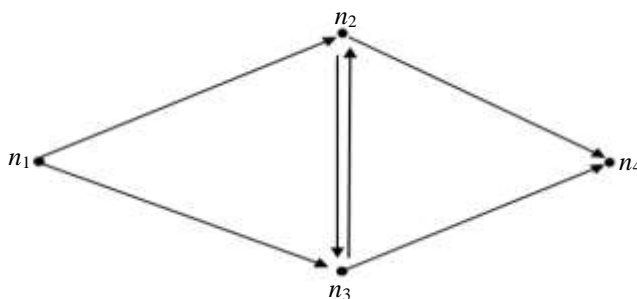


Fig. 3: Four-node network

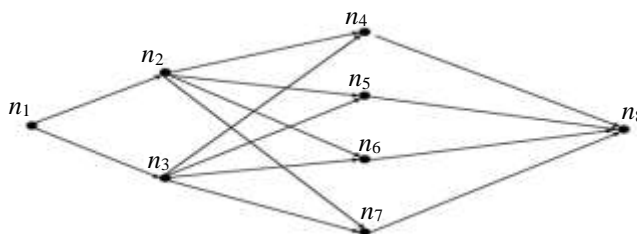


Fig. 4: Eight-node network

Table 1: The results for four-node network with  $d_1 = 3, d_2 = 2, w_1^1 = 1, w_1^2 = 1.5$

Run no.	The best M	Sum (M)	Reliability	Execution time (in seconds)
1	[6,4,4,6]	20	0.886015	0.05
2	[6,3,4,6]	19	0.879420	0.04
3	[6,3,3,6]	18*	0.785197	0.04

\*The minimum  $\mathbb{C}$  found

Table 2: The results for four-node network with  $d_1 = 2, d_2 = 3, w_1^1 = 1, w_1^2 = 1.5$

Run no.	The best M	Sum (M)	R (value)	Execution time (in seconds)
1	[7,3,4,7]	21*	0.754180	0.05
2	[7,4,4,7]	22	0.859764	0.05
3	[7,4,3,7]	21	0.754180	0.05

\*The minimum  $\mathbb{C}$  found

Table 3: The results for eight-node network with  $d_1 = 1, d_2 = 2, w_1^1 = 1, w_1^2 = 1.5$

Run no.	The best M	Sum (M)	Reliability	Execution time (in seconds)
1	[4,4,4,2,3,3,2,4]	26	0.431576	0.10
2	[4,1,4,4,3,2,4,4]	26	0.444284	0.07
3	[4,1,3,3,4,2,4,4]	25	0.332267	0.04
4	[4,2,4,4,4,4,1,4]	27	0.455874	0.07
5	[4,4,4,1,4,3,2,4]	26	0.461148	0.05
6	[4,3,4,3,1,3,4,4]	26	0.463200	0.08
7	[4,3,3,1,4,4,4,4]	27	0.439850	0.07
8	[4,4,3,4,1,3,1,4]	24*	0.457769	0.08
9	[4,1,3,4,2,4,3,4]	25	0.332445	0.08
10	[4,3,2,4,1,3,4,4]	25	0.318427	0.07

\*The minimum  $\mathbb{C}$  found

### Ten-Node Network

This network has ten nodes as shown in Fig. 5, it has seven minimal paths as follows (Chen and Lin,

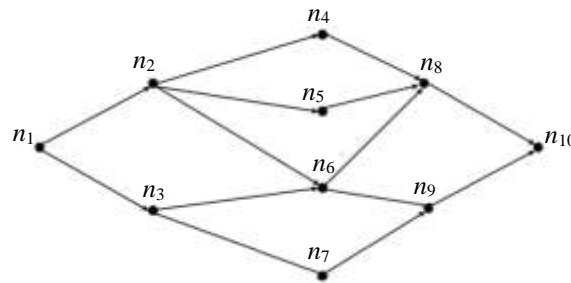
2010):  $mp_1 = \{n_1, n_2, n_4, n_8, n_{10}\}$ ,  $mp_2 = \{n_1, n_2, n_5, n_8, n_{10}\}$ ,  $mp_3 = \{n_1, n_2, n_6, n_8, n_{10}\}$ ,  $mp_4 = \{n_1, n_2, n_6, n_9, n_{10}\}$ ,  $mp_5 = \{n_1, n_3, n_6, n_8, n_{10}\}$ ,  $mp_6 = \{n_1, n_3, n_6, n_9, n_{10}\}$ ,  $mp_7 = \{n_1, n_3, n_7, n_9, n_{10}\}$ . The nodes'

probabilities,  $r_i = (0.99, 0.98, 0.97, 0.98, 0.98, 0.99, 0.97, 0.98, 0.97, 0.99)$ ,  $w_i^1 = 1$ ,  $w_i^2 = 1.5$ . Table 4 shows the best candidate  $M$  with the corresponding  $\mathbb{C}$ ,  $R_{1,2}$ , execution time.

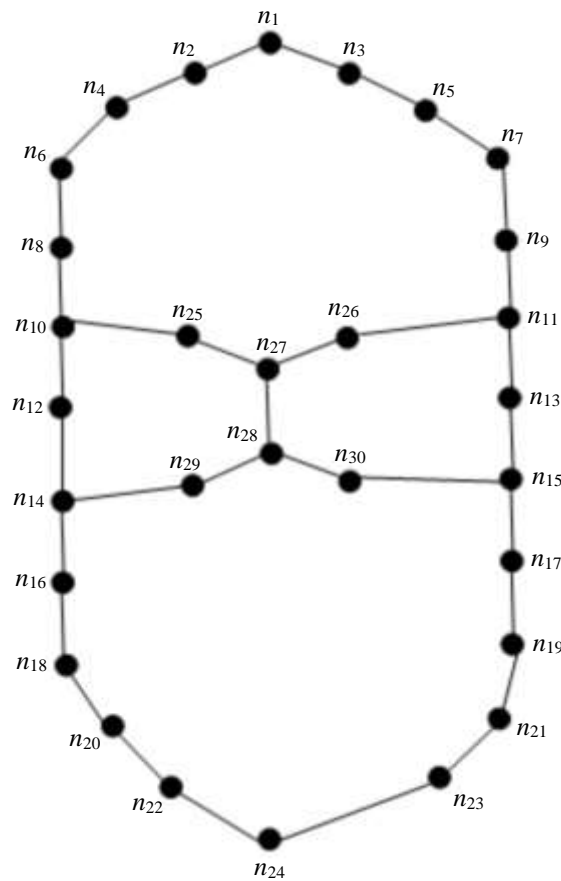
*Thirteen-Node Network*

The network has 30 nodes as shown in Fig. 6 and 6 minimal paths as follows:  $mp_1 = \{n_1, n_2, n_4, n_6, n_8, n_{10}, n_{12}, n_{14}, n_{16}, n_{18}, n_{20}, n_{22}, n_{24}\}$ ,  $mp_2 = \{n_1, n_3, n_5, n_7, n_9, n_{11}, n_{13}, n_{15}, n_{17}, n_{19}, n_{21}, n_{23}, n_{24}\}$ ,  $mp_3 = \{n_1, n_2, n_4, n_6, n_8, n_{10}, n_{25}, n_{27}, n_{28}, n_{29}, n_{14}, n_{16}, n_{18}, n_{20}, n_{22}, n_{24}\}$ ,

$mp_4 = \{n_1, n_2, n_4, n_6, n_8, n_{10}, n_{25}, n_{27}, n_{28}, n_{30}, n_{14}, n_{17}, n_{19}, n_{21}, n_{23}, n_{24}\}$ ,  $mp_5 = \{n_1, n_3, n_5, n_7, n_9, n_{11}, n_{26}, n_{27}, n_{28}, n_{30}, n_{15}, n_{17}, n_{19}, n_{21}, n_{23}, n_{24}\}$ ,  $mp_6 = \{n_1, n_3, n_5, n_7, n_9, n_{11}, n_{26}, n_{27}, n_{28}, n_{29}, n_{14}, n_{16}, n_{18}, n_{20}, n_{22}, n_{24}\}$ .  $r_i = (0.99, 0.88, 0.93, 0.91, 0.91, 0.89, 0.87, 0.91, 0.90, 0.90, 0.93, 0.92, 0.88, 0.88, 0.87, 0.92, 0.92, 0.93, 0.90, 0.90, 0.91, 0.87, 0.87, 0.99, 0.89, 0.93, 0.90, 0.93, 0.87, 0.89)$ . Table 5 shows the best candidate  $M$  with the corresponding  $\mathbb{C}$ ,  $R_{1,2}$ , execution time. Table 6 shows the best candidate  $M$  with the corresponding  $\mathbb{C}$ ,  $R_{1,1}$ , execution time.



**Fig. 5:** Ten-node network



**Fig. 6:** Thirty-node network

**Table 4:** The results for ten-node network with  $d_1 = 1, d_2 = 2, w_i^1 = 1, w_i^2 = 1.5$

Run no.	The best M	Sum (M)	Reliability	Execution time (in seconds)
1	[4,3,2,1,4,4,1,4,3,4]	30	0.867441	0.05
2	[4,4,3,1,2,4,4,3,3,4]	32	0.917677	0.07
3	[4,1,4,3,2,3,3,4,3,4]	31	0.910575	0.08
4	[4,4,4,1,4,1,4,4,3,4]	33	0.922117	0.04
5	[4,4,1,4,2,1,4,4,3,4]	31	0.914196	0.07
6	[4,2,4,2,3,4,1,1,4,4]	29*	0.910521	0.04
7	[4,2,4,1,4,4,1,4,3,4]	31	0.917152	0.07
8	[4,2,4,2,2,4,2,2,4,4]	30	0.912575	0.04
9	[4,4,1,4,3,1,1,4,4,4]	30	0.916173	0.04
10	[4,2,4,2,3,4,4,4,1,4]	32	0.912874	0.07

\*The minimum  $\mathbb{C}$  found

**Table 5:** The results for Fig. 6 network with  $d_1 = 2, d_2 = 2, w_i^1 = 1, w_i^2 = 1$

Run no.	The best M	Sum (M)	Reliability	Execution time (in seconds)
1	[4,4,3,3,4,2,2,4,4,4,3,1,4,3,3,2,2,4,4,4,2,3,4,4,2,2,2,1,4,2]	90	0.218391	0.05
2	[4,4,4,3,3,3,2,3,3,4,1,2,4,3,3,3,4,3,4,4,2,4,4,4,1,2,4,3,3,1]	92	0.166409	0.05
3	[4,4,3,3,3,3,4,1,4,2,4,4,4,4,3,4,4,3,2,4,4,2,3,4,3,2,1,3,4,2]	95	0.204131	0.05
4	[4,3,4,4,4,2,4,4,3,4,3,2,4,2,4,3,3,2,4,4,3,1,3,4,3,2,2,2,2,4]	93	0.216591	0.05
5	[4,2,4,4,4,3,4,4,4,2,4,1,3,4,3,2,3,4,2,3,4,4,4,4,1,1,3,2,3,4]	94	0.446587	0.06
6	[4,4,2,2,4,2,4,3,3,4,3,1,1,3,4,2,2,3,4,3,3,2,4,2,4,4,3,2,3]	88	0.194756	0.05
7	[4,1,3,2,3,3,3,2,3,1,4,1,4,3,3,4,4,4,4,3,3,3,2,4,1,4,3,3,4,4]	90	0.140628	0.05
8	[4,4,2,2,4,3,3,2,4,2,4,2,1,4,3,4,4,4,4,4,3,3,2,4,3,1,4,2,4,4]	94	0.259161	0.06
9	[4,3,3,4,3,2,3,4,4,4,4,2,1,3,2,3,3,3,4,4,1,4,2,4,2,2,4,4,4,4]	94	0.220534	0.05
10	[4,2,3,1,4,4,4,3,3,3,4,1,4,3,3,3,4,2,3,3,4,1,3,4,1,2,2,3,4,1]	86*	0.101526	0.05

\*The best value for  $\mathbb{C}$

**Table 6:** The results for Fig. 6 network with  $d_1 = 1, d_2 = 1, w_i^1 = 1, w_i^2 = 1.5$

Run no.	The best M	Sum (M)	Reliability	Execution time (in seconds)
1	[3,4,4,4,3,4,3,4,4,4,3,4,4,2,3,3,2,1,3,3,2,1,3,2,2,2]	87	0.469482	0.05
2	[3,4,3,2,2,4,4,3,2,4,3,3,4,4,4,4,3,2,2,3,3,4,3,3,1,3,2,3,4,4]	93	0.740452	0.05
3	[3,3,4,4,4,3,2,2,1,4,2,4,4,2,3,3,4,3,1,3,1,3,4,3,4,4,4,1,1,2]	86	0.358579	0.05
4	[3,3,4,4,3,2,2,1,4,1,2,4,4,2,3,4,3,3,4,3,3,4,3,4,1,2,1,4,1]	85	0.429503	0.05
5	[3,2,3,3,4,4,1,4,4,3,3,4,1,3,3,4,3,2,1,4,3,3,2,3,4,1,4,1,4,2]	86	0.406832	0.06
6	[3,4,3,4,2,4,2,4,4,3,2,1,4,4,3,2,4,2,4,2,4,2,2,3,4,4,4,4,3,2]	93	0.688758	0.05
7	[3,2,3,2,4,4,4,3,2,2,4,3,4,4,4,1,4,3,2,1,3,2,4,3,1,4,3,3,3,2]	87	0.453717	0.06
8	[3,4,3,3,1,4,4,3,1,2,4,2,2,2,4,4,3,2,1,3,3,3,3,3,1,2,2,4,1,3]	80*	0.277095	0.06
9	[3,4,3,3,4,2,3,2,4,3,3,2,4,4,1,4,1,4,1,3,3,3,3,2,4,3,2,4,3]	88	0.586286	0.05
10	[3,3,2,3,2,4,4,4,3,2,4,3,4,2,3,2,4,3,4,3,4,4,3,3,2,2,3,3,1,3]	90	0.764796	0.05

\*The best value for  $\mathbb{C}$

## Conclusion

The research studied the robust design problem in the case of tow-commodity ( $d_1, d_2$ ). Each node's capacity ranges from 1 to  $d$ , where  $d = (w_i^1 d_1) + (w_i^2 d_2)$ . The capacity of each critical node equals to  $d$  to avoid the reliability to drop to zero. The proposed GA-based algorithm succeeded to solve the robust design problem for a flow network with multiple demands. The obtained solution is distinguished by the minimum sum of the assigned capacities and the maximum reliability value.

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## Author's Contributions

All authors are equally contributed in this work and the article.

## Ethics

Authors confirm that this manuscript has not been published elsewhere and that no ethical issues are involved.

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