International Journal of Research in Nursing, 2013, 4 (1), 1-13
ISSN: 1949-0194

doi:10.3844/ijrnsp.2013.1.13 Published Online 4 (1) 2013 (http://www.thescipub.com/ijrn.toc)

Odds to Quicken Reporting Already Delayed Cases: Acquired Immune Deficiency Syndrome Incidences are Illustrated

Ramalingam Shanmugam

School of Health Administration, Texas State University-San Marcos, TX 78666, USA

Received 2012-08-24, Revised 2012-08-29; Accepted 2013-04-01

ABSTRACT

Delayed reporting in a medical system complicates efforts to estimate the number of cases that occurred in a time period. A case in point is the government's difficulty to estimate the number of Acquired Immune Deficiency Syndrome (AIDS) cases. The reporting delays are not intentional but are ongoing due to changing Federal regulations or medical definitions of the case like AIDS. To simplify the complications, this article approaches by modifying the geometric distribution. To be specific, let $0<1-\theta<1$ is a chance for a case (like AIDS) to be reported in the same time period of its occurrence to a (Federal or other) agency. If the reporting is missed in its occurrence time period, the case gets reported in a next or later time period. Let Y be the number of time periods skipped until its reporting. In this process, the reporting probability in a current period is chained with that of past period with an "odds of quickening" to report. The implication and significance of "quickening odds" are investigated and explained in this article, using the AIDS data with delayed reporting.

Keywords: Geometric Distribution, Survival Function, Medical System's Memory, Nuisance Parameter

1. INTRODUCTION

Who might have guessed in year 1981 that more than 45 million people would have died and another estimated 75 million people would have suffered worldwide with "Acquired Immune Deficiency Syndrome (AIDS)"? What is the genesis of AIDS? On June 5, 1981, the Center for Disease Control (CDC) first detailed the biopsy of "5 young men with a rare pneumonia". After failed immune system, their vital $CD4^+$ cells were invaded by viruses, bacteria, fungi and parasites. CDC (1985) and Chamberland *et al*. (1985) for details. The virus was detected by a Polymerase Chain Reaction (PCR). Later in June, 1982 the CDC (1985) announced that the world faced "a new, deadly sexually transmitted disease". A month later, the CDC coined the name: "Acquired Immune Deficiency Syndrome (AIDS)" to refer this illness.

The World Health Organization (WHO) estimated that about 33.4 million people were suffering with AIDS and two million people (including 330,000 children) died in 2009 alone. The AIDS has become a major deadly human illness in many parts of the world. A scary fact is that AIDS is spreading. The AIDS diagnosis is based on clinical symptoms which appear to vary with lag effects since an initial infection. The medical community periodically debates and perfects the definition of clinical symptoms and recommends that a person with the virus should be declared as an AIDS case only after the illness progressed enough to pass through benchmarks determined by the CDC (1985). The Federal government regulated that the laboratory evidence of the virus should no more be mandated to report an AIDS case. This cautionary federally imposed tedious approach causes an unavoidable reporting delay of AIDS cases. The CDC (1985) mentions that while 42,670 AIDS cases were diagnosed by physicians as of 31 March 1987 but only 33,350 of them were actually reported. Some AIDS cases are never reported while others are reported in any of the subsequent sixteen quarters (Hay and Wolak (1994) for data and their details).

The reporting delays result in practical difficulties to estimate the actual number of AIDS cases. Harris (1990) noticed that the reporting delays of even 0.6 months shifted the frequency trend of AIDS cases to the right and consequently, the estimated AIDS cases fell far

below the actual number. DeGruttola *et al*. (1992) reported that the number pediatric AIDS cases in New York City were under-estimated because of reporting delays which changed over the chronological time in a non-stationary manner. Lindsey (1996) considered bivariate intensity functions of non-stationary Poisson processes and a non-parametric methodology to undo the under-estimation of AIDS cases due to reporting delays. Pagano *et al*. (1994) developed a regression methodology to make an adjustment to an under-estimate of the completely unobservable actual number of AIDS cases because of reporting delays. Bacchetti (1996) identified that the 1993 re-definition of AIDS caused reporting delays and also disrupted the interpretations of the death trend of AIDS cases. Gebhardt *et al*. (1998) noticed based on a Bayesian generalized linear model on reverse-time hazards that many industrial countries including Switzerland and Spain incurred significant deaths because of AIDS but it was understood only much later because of reporting delays. Tabnak *et al*. (2000) developed a change-point model to correct a biased estimate of AIDS cases because of reporting delays. Cui (1999) developed a nonparametric method to analyze Australian left-censored and right-truncated AIDS data and estimated the impact of the reporting delays.

The reporting delays occur in other topics also. Lawless (1994) mentioned that reporting delays occurred in insurance claims and provided a method to model the random temporal fluctuations to compensate for the under-reported claims. MacArthur *et al*. (1985) traced the source of under-reporting of tumor and other cancers and found that the hospitals rather than the patients cause reporting delays. Clegg *et al*. (2002) pointed out that reporting delays occur in cancer reporting medical after informing that the reporting delays actually confused the health officials to comprehend the cancer incidence trend as they contained estimation errors with downwardly biased cancer incidence trends and provided an approach with an appropriate methodology to obtain reporting-error-adjusted cancer incidence rate. Midthune *et al*. (2005) provided a methodology to make adjustments for an accurate cancer incidence rate in general and melanoma cancer in particular in the U.S. Zou *et al*. (2009) provided a methodology to capture the effect of reporting year on delay modeling of cancer incidence.

All above mentioned reasons motivate the importance and necessity for an additional statistical methodology to estimate number of cases like AIDS with reporting delays. A new methodology is pursued in this article by modifying geometric distribution to suit the reality in reporting medical system. This modified probability pattern is named Oscillating Geometric Odds Distribution (OGOD). Benefits include not only a way to estimate the actual number of AIDS cases in a given time period but also offer a statistical methodology to assess the significance of the estimated "odds of quickening" to improve reporting of an already delayed reporting. This methodology helps health administrators to prepare budgets and policies based on a better estimate of the cases like AIDS. Healthcare policies emerge from facts and perceptions. Fan (2004) outline the society's fears and phobias because of threat from AIDS illness. Understanding the AIDS prevalence using OGOD might help to reduce the psychological, social, economic fears or to combat the health insurance industry's denials to deserving applicants with AIDS symptoms. The reporting delay is not unique to AIDS illness alone and is suspected to exist in other illnesses as well. The contents of this article are versatile enough to explain the consequences of reporting delays in engineering, economics, public health, business or other disciplines as well.

The statistical properties of OGOD are derived in section 2. In section 3, they are illustrated with the data about reporting delays of AIDS cases in Hay and Wolak (1994). The last section 4 contains conclusive thoughts and recommendations.

1.1. Oscillating Geometric Odds Distribution

Let $0<1-\theta<1$ be a probability of reporting a case (like AIDS) in the same period of its occurrence. Delayed reporting in a medical or other system complicates efforts to estimate the actual number of cases that occurred in a time period. Fan (2004) for details. To resolve this difficulty, this article approaches by modifying geometric distribution as follows. The odds of reporting a case in the same period of its occurrence are $\text{odds}_\theta = \frac{\theta}{1-\theta}$. Then, a non-negative integer random variable Y, denoting the number of skipped time periods until its reporting follows a reparametrized geometric distribution Equation 1:

$$p(y|\theta) = \Pr[Y = y] = \frac{(\text{odds}_\theta)^y}{(1+\text{odds}_\theta)^{y+1}} \quad (1)$$

where, $y = 0,1,2,...,\infty; 0 < \theta < 1$. The probability of reporting a case in the same period of its occurrence is $\Pr[Y = 0] = \frac{1}{(1+\text{odds}_\theta)}$. Interestingly, the mean $\mu_\theta = \text{odds}_\theta$. A modification of (1) is necessary to suit an ongoing delayed reporting with an "odds of quickening", $\text{odds}_\phi = \frac{\phi}{1-\phi} > 0$. Consequently, the reporting chance

undergoes a fluctuation in the coming time periods. The fluctuations contradict the memory less property of geometric distribution (1). What is memory less property? It means the conditional probability of reporting a case in a time period given it has not been reported so far since its occurrence equals its unconditional probability of reporting in its period of occurrence itself. This is translated in probability terminology below in (2). Note from (1) that Equation 2:

$$\begin{aligned}&\Pr[Y \geq m+t \mid Y \geq m]\\&=\frac{\Pr[Y \geq m+t \cap Y \geq m]}{\Pr[Y \geq m]}\\&=\left(\frac{odds_\theta}{1+odds_\theta}\right)^m=\Pr[Y \geq m]\end{aligned} \tag{2}$$

The ongoing delays create a memory in a reporting medical system. Note that $odds_\theta = 0$ when the case is reported in the same period of its occurrence and the "odds of quickening" is obsolete. Otherwise, the $odds_\phi$ is fused into the reporting probability in a chained manner like:

$$\begin{aligned}&pr(Y=y)\\&=\begin{cases}\theta^y[pr(Y=y-1)+\frac{\phi}{1-\phi}] & \text{if } y=1,2,.....\\ 1-\theta & y=0\end{cases}\end{aligned}$$

This modification results in a new probability pattern Equation 3:

$$\begin{aligned}&p(y \mid \phi,\theta)=\Pr[Y=y]=\\&\frac{[1+(odds_\phi)(1+odds_\theta)y](odds_\theta)^y}{[1+(odds_\phi)(odds_\theta)(1+odds_\theta)](1+odds_\theta)^{y+1}}\end{aligned} \tag{3}$$

with $y = 0,1,2,\ldots,\infty$; $0<\theta<1, 0<\phi<1$.

Is expression (3) a bona fide probability distribution? The answer is affirmative. Trivially, the expression (3) is non-negative. The sum of their values equals one as it is shown below:

$$\begin{aligned}&\sum_{y=0}^{\infty}p(y \mid \phi,\theta)=\frac{[1+(1+odds_\theta)odds_\phi y](odds_\theta)^y}{[1+odds_\theta(1+odds_\theta)odds_\phi](1+odds_\theta)^{y+1}}\\&=\frac{[(1-\theta)+(\frac{\phi}{1-\phi})(1-\theta)\theta\partial_\theta(1-\theta)^{-1}]}{[(1-\theta)+(\frac{\phi}{1-\phi})(\frac{\theta}{1-\theta})]}=1\end{aligned}$$

where, ∂_θ denotes the derivative with respect to θ. The expression (3) is named "Oscillating Geometric Odds Distribution (OGOD)".

In the absence of "odds of quickening" to report in a system (that is, $\phi = 0$), the OGOD (3) reduces to the geometric distribution in (1) as a particular case. Otherwise, when all cases are reported in the same period, note that Y = 0. The reporting medical system has no lag with a probability Equation 4:

$$\begin{aligned}&p(Y=0 \mid \phi \neq 0,\theta)=[1+odds_\theta]^{-1}\\&[1+(odds_\theta)(1+odds_\theta)(odds_\phi)]^{-1}\end{aligned} \tag{4}$$

The probability for a reporting medical system to be busy with a lag of cases to report is Equation 5:

$$\begin{aligned}&p(Y \geq 1 \mid \phi \neq 0,\theta)=1-[1+odds_\theta]^{-1}\\&[1+(odds_\theta)(1+odds_\theta)(odds_\phi)]^{-1}\end{aligned} \tag{5}$$

The odds for a case to be reported in a medical system with a lag is Equation 6a,b:

$$odds_{\phi,\theta}=\frac{\Pr[Y \geq 1]}{\Pr[Y=0]} \approx (odds_\theta)a_{\phi,\theta} \tag{6a}$$

Where:

$$a_{\phi,\theta} \approx 1+\phi(1+odds_\theta)^2 \tag{6b}$$

Signifies an impact of "odds of quickening" on lag. A Taylorization is used to obtain (6a) and it is:

$$f(\phi,\theta) \approx f(\phi=0,\theta)+\phi[\partial_{\phi=0}f(\phi,\theta)]$$

where, $\partial_{\phi=0}f(\phi,\theta)]$ is the derivative of a function f (ϕ = θ)] evaluated at $\phi = 0$. Now, statistical properties of the OGOD are discussed. First, the mean is derived. That is Equation 7:

$$\begin{aligned}&\mu_{\phi,\theta}=E(Y \mid \phi,\theta)\\&[(1-\theta)^2\theta\sum_{y=0}^{\infty}y\theta^{y-1}+(\frac{\phi}{1-\phi})(1-\theta)\\&=\frac{\{\theta^2\sum_{y=0}^{\infty}y(y-1)\theta^{y-2}+\theta\sum_{y=0}^{\infty}y\theta^{y-1}\}]}{[(1-\theta)+(\frac{\phi}{1-\phi})(\frac{\theta}{1-\theta})]}\\&=\mu_\theta[1+\frac{(odds_\phi)(1+(odds_\theta)^2)}{\{1+(odds_\phi)(odds_\theta)(1+odds_\theta)\}}]\\&\approx[odds_\theta][1+\phi(1+odds_\theta)^2].\end{aligned} \tag{7}$$

Interestingly, the mean in an absence of "odds of quickening" to report delayed cases (that is, $odds_\phi = 0$ or equivalently, $\phi = 0$) is

$\mu_{\phi=0,\theta} = E(Y|\phi = 0,\theta) = \mu_\theta = \frac{\theta}{1-\theta} = odds_\theta$. Hence, the mean, (7) is viewed as $\mu_{\phi,\theta} = \mu_\theta a_{mean,\phi,\theta}$ where Equation 8:

$$\begin{aligned} & a_{mean,\phi,\theta} \\ & = [1 + \frac{(odds_\phi)(1+odds_\theta)^2)}{\{1+(odds_\phi)(odds_\theta)(1+odds_\theta)\}}] \\ & \approx odds_{\phi,\theta} \\ & \approx [odds_\theta][1+\phi(1+odds_\theta)^2] \end{aligned} \quad (8)$$

Is an impact of "odds of quickening" on mean. The impact values could be compared over the years to get a clue on how the delayed reporting has improved. This knowledge is useful to health administrators. It is easy to see that Equation 9a-d:

$$(1-\theta)^{-1} = 1+\theta+\theta^2+..... = \sum_{y=0}^{\infty}\theta^y \quad (9a)$$

$$(1-\theta)^{-2} = 1+2\theta+3\theta^2+.. = \frac{1}{\theta}\sum_{y=0}^{\infty} y\theta^y \quad (9b)$$

$$2\theta^2(1-\theta)^{-3} + \theta(1-\theta)^{-2} = \sum_{y=0}^{\infty} y^2\theta^y \quad (9c)$$

$$\begin{aligned} & 6\theta^3(1-\theta)^{-4} + 6\theta^2(1-\theta)^{-3} + \theta(1-\theta)^{-2} \\ & = \sum_{y=0}^{\infty} y^3\theta^y \end{aligned} \quad (9d)$$

The dispersion $\sigma^2_{\phi,\theta}$ of the OGOD is obtained using the relations in (9a through 9d). After algebraic simplifications, it turns out to be Equation 10:

$$\sigma^2_{\phi,\theta} \approx (odds_\theta)(1+odds_\theta)|1-\phi(odds_\theta)| \quad (10)$$

In the absence of "odds of quickening" to report (that is, ϕ = 0), expression (10) yields dispersion $\sigma^2_{\phi=0,\theta} = (odds_\theta)(1+odds_\theta)$ of the geometric distribution (1). Hence, dispersion (10) is viewed as $\sigma^2_{\phi,\theta} \approx \sigma^2_{\phi=0,\theta} a_{variance,\phi,\theta}$ where Equation 11:

$$a_{dispersion,\phi,\theta} = |1-\phi(odds_\theta)| \quad (11)$$

Portrays an impact of "odds of quickening" to report on dispersion. Next, the survival function:

$$\bar{G}_{\phi,\theta}(r+1) = Pr[Y \geq r+1|\phi,\theta]$$

For the OGOD (3) is derived in terms of the F-distribution. The table for F-distribution is popularly available. The incomplete beta function in (12) is indeed F-distribution. It is easy to see that Equation 12:

$$\begin{aligned} & \sum_{y=r+1}^{\infty}(1-\theta)\theta^y = (r+1)\int_{1-\theta}^{1}(1-y)^r dy \\ & = (\frac{odds_\theta}{1+odds_\theta})^{r+1} = IB_\theta(r+1,1) \\ & = Pr[F_{2(r+1),2} \leq \frac{(odds_\theta)}{(r+1)}] \end{aligned} \quad (12)$$

Hence, the survival function is Equation13:

$$\begin{aligned} & \bar{G}_{\phi,\theta}(r+1) = Pr[Y \geq r+1|\phi,\theta] \\ & = \sum_{y=r+1}^{\infty} p(y|\phi,\theta) \\ & = \frac{[1+(1+odds_\theta)odds_\phi y](odds_\theta)^y}{[1+odds_\theta(1+odds_\theta)odds_\phi](1+odds_\theta)^{y+1}} \\ & \approx Pr[F_{2(r+1),2} \leq \frac{(odds_\theta)}{(r+1)}] + \phi(1+odds_\theta) \\ & \{(r+1)Pr[F_{2r,2} \leq \frac{(odds_\theta)}{r}] \\ & -\mu_\theta Pr[F_{2(r+1),2} \leq \frac{(odds_\theta)}{(r+1)}]\}. \end{aligned} \quad (13)$$

The survival function (13) could be viewed as $\bar{G}_{\phi,\theta}(r+1) = \bar{G}_{\phi=0,\theta}(r+1)a_{survival,\phi,\theta}$ where an impact of "odds of quickening" to report on survival function is Equation 14:

$$\begin{aligned} & a_{survival,\phi,\theta} = 1+\phi(1+odds_\theta) \\ & \{(r+1)\frac{Pr[F_{2r,2} \leq \frac{(odds_\theta)}{r}]}{Pr[F_{2(r+1),2} \leq \frac{(odds_\theta)}{(r+1)}]} - (odds_\theta)\} \end{aligned} \quad (14)$$

By substituting r = 0 in (13), it yields the chance for a lag to exist. That is Equation 15:

$$\begin{aligned} & \bar{G}_{\phi,\theta}(1) = Pr[Y \geq 1|\phi,\theta] \approx Pr[F_{2,2} \leq (odds_\theta)] \\ & +\phi(1+odds_\theta)\{1-(odds_\theta)Pr[F_{2,2} \leq (odds_\theta)]\} \end{aligned} \quad (15)$$

In the absence of "odds of quickening" to report (that is, ϕ = 0), expression (15) reduces to Equation 16:

$$\begin{aligned} & \bar{G}_{\phi=0,\theta}(1) = Pr[Y \geq 1|\phi = 0,\theta] \\ & \approx Pr[F_{2,2} \leq (odds_\theta)] \end{aligned} \quad (16)$$

So, what is an impact of "odds of quickening" to report on medical system to be "busy"? This is implied in the relation $\bar{G}_{\phi,\theta}(1) = \bar{G}_{\phi=0,\theta}(1)a_{busy,\phi,\theta}$ where the level of busy is Equation 17:

$$a_{busy,\phi,\theta}(1) = 1 + \phi\{1 + (odds_\theta)\} (\{Pr[F_{2,2} \le (odds_\theta)]\}^{-1} - (odds_\theta)\}) \quad (17)$$

and it portrays an impact of "odds of quickening" to report on busy. Now, a discussion on how much a memory is created in a reporting medical system because of the lag. Recall that the geometric distribution (1) is known to possess a memory less property as shown in (2). From OGOD (3), note that:

$$\begin{aligned} &Pr[(Y \ge m + t \,|\, Y \ge m)|\phi,\theta] \\ &= \frac{Pr[(Y \ge m + t \cap Y \ge m)|\phi,\theta]}{Pr[Y \ge m|\phi,\theta]} \\ &= m_{\phi,\theta}\, Pr[(Y \ge t||\phi,\theta] \end{aligned}$$

where a reporting medical-system's memory is Equation 18:

$$\begin{aligned} m_{\phi,\theta} = {} & \frac{\begin{array}{l}[1 + (odds_\phi)\{1 + (odds_\theta)\} \\ \{m + t + (odds_\theta)\}] \\ [1 + (odds_\phi)(odds_\theta)\{1 + (odds_\theta)\}] \\ [1 + (odds_\phi)\{1 + (odds_\theta)\} \\ \{m + (odds_\theta)\}] \\ [1 + (odds_\phi)\{1 + (odds_\theta)\} \\ \{t + (odds_\theta)\}]\end{array}}{\begin{array}{l}\{Pr[F_{2(m+t),2} \le \frac{(odds_\theta)}{(m+t)}] \\ +\phi(1 + (odds_\theta))\{(m+t)\, Pr[F_{2r,2} \le \frac{(odds_\theta)}{m+t-1}]\end{array}} \\ m_{\phi,\theta} = {} & \frac{\begin{array}{l}-(odds_\theta)\, Pr[F_{2(m+t),2} \le \frac{(odds_\theta)}{m+t}]\} \\ \{Pr[F_{2m,2} \le \frac{(odds_\theta)}{m}] + \phi\{1 + (odds_\theta)\} \\ \{m\, Pr[F_{2r,2} \le \frac{(odds_\theta)}{m-1}] \\ -\mu_\theta\, Pr[F_{2m,2} \le \frac{(odds_\theta)}{m}]\} \\ \{Pr[F_{2m,2} \le \frac{(odds_\theta)}{t}] \\ +\phi\{1 + (odds_\theta)\}\{t\, Pr[F_{2r,2} \le \frac{(odds_\theta)}{t-1}] \\ -(odds_\theta)\, Pr[F_{2t,2} \le \frac{(odds_\theta)}{t}]\}\end{array}}{\begin{array}{l}\{Pr[F_{2m,2} \le \frac{(odds_\theta)}{m}] + \phi\{1 + (odds_\theta)\} \\ \{m\, Pr[F_{2r,2} \le \frac{(odds_\theta)}{m-1}] \\ -\mu_\theta\, Pr[F_{2m,2} \le \frac{(odds_\theta)}{m}]\} \\ \{Pr[F_{2m,2} \le \frac{(odds_\theta)}{t}] \\ +\phi\{1 + (odds_\theta)\}\{t\, Pr[F_{2r,2} \le \frac{(odds_\theta)}{t-1}] \\ -(odds_\theta)\, Pr[F_{2t,2} \le \frac{(odds_\theta)}{t}]\}\end{array}} \end{aligned} \quad (18)$$

In the absence of "odds of quickening" to report (that is, $\phi = 0$ or $odds_\phi = 0$), expression (18) reduces to baseline value one, confirming the memory less property as stated in (2) for geometric distribution (1). Hence, a theorem is stated.

Theorem 1:

The chance mechanism which is governed by the oscillating geometric odds distribution (3) has a finite memory $m_{\phi,\theta}$ in (18).

Now, the Maximum Likelihood Estimate (MLE) of the parameters ϕ and θ are obtained. A reason for choosing the MLE is that it is invariant. The MLE helps to perform a data analysis. Consider a random sample y_1, y_2, y_3,...., y_n from OGOD (3). Then, the log likelihood function is Equation 19:

$$\begin{aligned} &\ln L(\phi,\theta) = -n \ln[1 - \theta + odds_\phi odds\theta] \\ &+\sum_{i=1}^{n} \ln[\{(1-\theta)^2 + odds_\phi(1-\theta)y_i\}\theta^{y_i}] \\ &\approx -n \ln\{1 + (odds_\theta)\} + n\phi(odds_\theta)\{1 + (odds_\theta)\} \\ &+n\bar{y} \ln\{\frac{(odds_\theta)}{1 + (odds_\theta)}\} + n\phi\bar{y}\{1 + (odds_\theta)\} \end{aligned} \quad (19)$$

The MLE $\hat{\phi}_{mle}$ and $\hat{\theta}_{mle}$ are the solutions of the score functions $\partial_\phi \ln L(y_1, y_2, y_3, ..., y_n, \theta, \phi) = 0$ and $\partial_\theta \ln L(y_1, y_2, y_3, ..., y_n, \theta, \phi) = 0$. Both score functions are nonlinear. The non-linearity is eased by their Taylor's series expansion. They result in the MLEs in (20) and (21) after algebraic simplifications. They are Equation 20 and 21:

$$\hat{\phi}_{mle} \approx \frac{\left|\bar{y}(1+\bar{y}) - s_y^2\right|}{\bar{y} + \left|\bar{y}(1+\bar{y}) - s_y^2\right|} \quad (20)$$

And:

$$\hat{\theta}_{mle,\hat{\phi}_{mle}} \approx (1 - \frac{\hat{\phi}_{mle}\bar{y}}{1+\bar{y}})\frac{\bar{y}}{(1+\bar{y})} \quad (21)$$

In the absence of "odds of quickening" to report, note that $\bar{y}(1+\bar{y}) \to s_y^2$ and expression (21) reduces to $\hat{\theta}_{mle,\hat{\phi}_{mle}=0} = \frac{\bar{y}}{(1+\bar{y})}$ pertaining to geometric distribution (1). In other words, geometric distribution (1) possesses a balance, $s_y^2 = \bar{y}(1+\bar{y})$ between the sample variance s_y^2 and the quadratic expression $\bar{y}(1+\bar{y})$. This property is named

"dispersion balance" and it exists in the absence of "odds of quickening" to report. Otherwise, there exists either "dispersion underbalance" with $s_y^2 < \bar{y}(1+\bar{y})$ or "dispersion overbalance" with $s_y^2 > \bar{y}(1+\bar{y})$ as a characteristic property of OGOD (3). The statistics community has been debating about under or over dispersion in data. However, the MLE of the mean in (7) is Equation 22:

$$\hat{\mu}_{\hat{\phi}_{mle},\theta} \approx \left(\frac{1-\hat{\phi}_{mle}\frac{\bar{y}}{1+\bar{y}}}{1+\hat{\phi}_{mle}\frac{\bar{y}^2}{1+\bar{y}}}\right)\left(1+\frac{\hat{\phi}_{mle}[1+\bar{y}]^2}{[1+\hat{\phi}_{mle}\frac{\bar{y}^2}{1+\bar{y}}]^2}\right)\bar{y} \quad (22)$$

which reduces to $\hat{\mu}_{\hat{\phi}_{mle}=0,\theta} = \bar{y}$ when $\hat{\phi}_{mle} \to 0$ of the geometric distribution as a particular case in the absence of "odds of quickening" to report.

A health administrator is interested in a "ratio" $R = \frac{Y}{T-Y} > m$ where m and T are respectively a specified threshold level and a total number of reported cases in a year. The ratio is the odds of the number of reported versus unreported number cases in a year. The epidemiologists are fond of this kind of odds. To make a probability assessment about the odds, the expected and variance values of R are needed. To find them, the formulas Equation 23a,b:

$$E\left(\frac{W}{V}\right) = \frac{E_w}{E_v}\left(1+\frac{V_v}{E_v^2}-\frac{2\,cov[W,V]}{E_w E_v}\right) \quad (23a)$$

$$Var\left(\frac{W}{V}\right) = \frac{E_w^2}{E_v^2}\left(\frac{V_w}{E_w^2}+\frac{V_v}{E_v^2}-\frac{2\,cov[W,V]}{E_w E_v}\right) \quad (23b)$$

are used (Stuart and Ord (1994) for details), where E_j and V_j denote the mean and variance respectively of j = W or V. That is Equation 24 and 25:

$$E(\Re) = E\left(\frac{Y}{T-Y}\right) = \frac{\mu_{\phi,\theta}}{[T-\mu_{\phi,\theta}]}\left(1+\frac{\sigma_{\phi,\theta}^2}{[T-\mu_{\phi,\theta}]^2}-\frac{2\,cov[Y,T-Y]}{\mu_{\phi,\theta}[T-\mu_{\phi,\theta}]}\right) = \frac{\mu_{\phi,\theta}}{[T-\mu_{\phi,\theta}]}\left(1+\frac{\sigma_{\phi,\theta}^2[2T-\mu_{\phi,\theta}]}{\mu_{\phi,\theta}[T-\mu_{\phi,\theta}]^2}\right) \quad (24)$$

And:

$$Var(\Re) = Var\left(\frac{Y}{T-Y}\right) = \frac{T^2\sigma_{\phi,\theta}^2}{[T-\mu_{\phi,\theta}]^4} \quad (25)$$

Hence, $Pr[\Re > m] \approx 1-\Phi_z\left(\frac{m-E[\hat{\Re}_{mle}]}{\sqrt{Var[\hat{\Re}_{mle}]}}\right)$ where Φ_z (p) is the cumulative area under the standard normal distribution up to a percentile p, the MLEs $E[\hat{\Re}_{mle}]$ and $Var[\hat{\Re}_{mle}]$ are computed using (23), (24), (25), (20) and (21).

A healthcare administrator often ponders over a question: Do the data give a clue about the absence or insignificant level of "odds of quickening"? An answer to this question requires a hypothesis testing methodology. For this purpose, the Wald (1943) likelihood ratio concept is invoked in this article. According to Wald's concept, to test the null hypothesis $H_o: \phi = 0$ against an alternative hypothesis $H_1: \phi = \phi^* \neq 0$, the log-likelihood ratio is Equation 26:

$$-\ln\Lambda_{\phi^*} = \ln L(\hat{\phi}_{mle},\hat{\theta}_{\hat{\phi},mle}) - \ln L(\phi^*,\hat{\theta}_{\phi^*,mle}) \approx n[\{\bar{y}\ln\hat{\theta}_{\hat{\phi},mle}+\ln(1-\hat{\theta}_{\hat{\phi},mle})\}-\hat{\phi}_{mle}\{\frac{1-\bar{y}}{1-\hat{\theta}_{\hat{\phi},mle}}\} -\{\bar{y}\ln\hat{\theta}_{\phi^*,mle}+\ln(1-\hat{\theta}_{\phi^*,mle})\}-\phi^*\{\frac{1-\bar{y}}{1-\hat{\theta}_{\phi^*,mle}}\}] \quad (26)$$

Under the null hypothesis, $\hat{\theta}_{mle,\phi=0} = \frac{\bar{y}}{(1+\bar{y})}$, $\hat{\mu}_{\hat{\theta}_{mle,\phi=0}} = \frac{\hat{\theta}_{mle,\phi=0}}{1-\hat{\theta}_{mle,\phi=0}} = \bar{y}$ and hence Equation 27:

$$-\ln\Lambda_{\phi=0} \approx n[\hat{\phi}_{mle}\{\frac{1-\bar{y}}{1-\hat{\theta}_{\hat{\phi},mle}}\}-\{\bar{y}\ln(\frac{\hat{\theta}_{\hat{\phi},mle}(1+\bar{y})}{\bar{y}})\} +\ln\{(1-\hat{\theta}_{\hat{\phi},mle})(1+\bar{y})\}] \quad (27)$$

Which follows a non-central chi-squared distribution with one degrees of freedom (df) and the non-centrality parameter $\hat{\delta}_{\phi=0} = \hat{\phi}_{mle} / var(\hat{\phi}_{mle})$, where $var(\hat{\phi}_{mle})$ is a diagonal element in the inverse of the variance-covariance matrix of the MLEs. Stuart and Ord (1994) for definition and properties of the non-central chi

squared distribution. Recall that the variance-covariance matrix of the MLE of the parameters is the inverse of the information matrix $I = \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} -E(\partial^2_{\phi\phi} \ln L) & -E(\partial^2_{\phi\theta} \ln L) \\ -E(\partial^2_{\phi\theta} \ln L) & -E(\partial^2_{\theta\theta} \ln L) \end{bmatrix}$, with $a = -E(\partial^2_{\phi\phi} \ln L) = 0$,

$b = -E(\partial^2_{\phi\theta} \ln L) \approx n[2(\frac{\theta}{1-\theta}) + 1 - \mu_{\phi,\theta}] / (1-\theta)^2$ and

$c = -E(\partial^2_{\theta\theta} \ln L) \approx n[\frac{\mu_{\phi,\theta}}{\theta^2} + \frac{1}{(1-\theta)^2} - \frac{2\phi}{(1-\theta)^3} \{\mu_{\phi,\theta} - 2 - 3(\frac{\theta}{1-\theta})\}]$.

The determinant of the matrix I is D = |I| = $-b^2$ and $\text{var}(\hat{\phi}_{mle}) = b^{-1}$. Hence:

$$\hat{\delta}_{\phi=0} = \hat{\phi}_{mle} / \text{var}(\hat{\phi}_{mle}) \approx \frac{[\frac{|\bar{y}(1-\bar{y}) - s_y^2|}{\bar{y} + |\bar{y}(1-\bar{y}) - s_y^2|}](1 - \hat{\theta}_{mle,\hat{\phi}_{mle}})^2}{n[2(\frac{\hat{\theta}_{mle,\hat{\phi}_{mle}}}{1 - \hat{\theta}_{mle,\hat{\phi}_{mle}}}) + 1 - \hat{\mu}_{\hat{\phi}_{mle},\hat{\theta}_{\hat{\phi},mle}}]}$$

The non-central chi squared distribution with one df and non-centrality parameter δ approximately follows $(1 + \frac{\delta}{1+\delta})$ times a central chi squared distribution with $\frac{(1+\delta)^2}{(1+2\delta)}$ df (Stuart and Ord (1994) for details). This means that the null hypothesis $H_o : \phi = 0$ will be rejected in favor of an alternative hypothesis $H_o : \phi = \phi^* \neq 0$ when:

$$-\ln \Lambda_{\phi=0} > (1 + \frac{\hat{\delta}_{\phi=0}}{1 + \hat{\delta}_{\phi=0}}) \chi^2_{\frac{(1+\hat{\delta}_{\phi=0})^2}{(1+2\hat{\delta}_{\phi=0})}\text{df},\alpha}$$

where the right side is the critical value based on the $100(1-\alpha)^{th}$ percentile of the central chi squared distribution with $\frac{(1+\hat{\delta}_{\phi=0})^2}{(1+2\hat{\delta}_{\phi=0})}$ df and a significance level $\alpha \in (0, 1)$. The p-value for rejecting the null hypothesis in favor of an alternative hypothesis is Equation 28:

$$p - value = \Pr[\chi^2_{\frac{(1+\hat{\delta}_{\phi=0})^2}{(1+2\hat{\delta}_{\phi=0})}\text{df}} < \frac{-\ln \Lambda_{\phi=0}}{(1 + \frac{\hat{\delta}_{\phi=0}}{1 + \hat{\delta}_{\phi=0}})}] \quad (28)$$

The statistical power of the test statistic (28) is now examined with a selection of a specific attainable value for ϕ^* in the alternative hypothesis. The statistical power is the probability of accepting a true alternative hypothesis $H_1 : \phi = \phi^* \neq 0$. Under an alternative hypothesis, the minus log likelihood ratio follows a non-central chi-squared distribution with one df and non-centrality parameter:

$$\hat{\delta}_{j^*} = \frac{(\hat{j}_{mle} - j^*)}{\text{var}(\hat{j}_{mle})} \approx \frac{[\frac{|\bar{y}(1-\bar{y}) - s_y^2|}{\bar{y} + |\bar{y}(1-\bar{y}) - s_y^2|} - j^*](1 - \hat{\theta}_{mle,\hat{j}_{mle}})^2}{n[2(\frac{\hat{\theta}_{mle,\hat{j}_{mle}}}{1 - \hat{\theta}_{mle,\hat{j}_{mle}}}) + 1 - \hat{\mu}_{\hat{j}_{mle},\hat{\theta}_{mle,\hat{j}_{mle}}}]}$$

This non-central chi squared distribution with one df and non-centrality parameter $\hat{\delta}_{\phi^*}$ is approximately $(1 + \frac{\hat{\delta}_{\phi^*}}{1 + \hat{\delta}_{\phi^*}})$ times a central chi squared score with $\frac{(1+\hat{\delta}_{\phi^*})^2}{(1+2\hat{\delta}_{\phi^*})}$ df. That is Equation 29:

$$\text{Power} = \Pr[\chi^2_{\frac{(1+\hat{\delta}_{\phi^*})^2}{(1+2\hat{\delta}_{\phi^*})}\text{df}} < \frac{(1 + \frac{\hat{\delta}_{\phi=0}}{1 + \hat{\delta}_{\phi=0}})(-\ln \Lambda_{\phi=\phi^*}) \chi^2_{\frac{(1+\hat{\delta}_{\phi=0})^2}{(1+2\hat{\delta}_{\phi=0})}\text{df},\alpha}}{(1 + \frac{\hat{\delta}_{\phi^*}}{1 + \hat{\delta}_{\phi^*}})(-\ln \Lambda_{\phi=0})}] \quad (29)$$

1.2. Reporting Aids Cases for Illustration

The results of the section 2 are illustrated using delayed reporting of AIDS cases in Hay and Wolak (1994) as quoted in **Table 1**. The number, n of quarters in their delayed reporting during 1982 through 1990 ranged from 2 to 17. The Maximum Likelihood Estimates (MLE) of the parameters is displayed in **Table 2**. The high value of $\hat{\phi}_{mle}$ in its domain [0, 1] is indicative of the existence of "odds of quickening" to report. The estimate of the probability $\hat{\theta}_{mle}$ for a case to be delayed

without reporting in its occurrence period itself is high in its domain [0, 1] only in the beginning during 1982 but it has been decreasing over the years. The p-value for the null hypothesis $H_o: \phi = 0$ to be true is small enough to believe that "odds of quickening" to report indeed existed. The medical administrators had been quite consciously trying to quicken the reporting of already delayed cases. With the total, T number of AIDS cases, the power of accepting H_1: $\phi^* = 0.5$ using the given data is higher in a quarter during 1982 through 1990 (**Table 2**). The **Fig. 1** suggests that the chance for reporting an AIDS case at a later quarter had been increasing over the years though the total number of AIDS cases grew according to **Fig. 2**. As can be seen in **Fig. 3**, the chance for majority of the AIDS cases get reported in the same quarter of its occurrence oscillated over the years but it became phenomenal in the later time period. The odds of reporting an AIDS case in the same quarter of its occurrence increased more in the beginning than in later period during years 1982-1990 due to "odds of quickening" to report, according to the **Fig. 4**. The **Fig. 5** indicates that the impact of "odds of quickening" to report on busy medical system during years 1982-1990. Together, the MLE $\hat{\theta}_{mle,\hat{\phi}_{mle}=0} = \frac{\bar{y}}{(1+\bar{y})}$ and $odds_{\hat{\theta}_{mle,\hat{\phi}_{mle}=0}} = \bar{y}$ portray the probability to report an AIDS case in the absence of "odds of quickening". Notice in **Fig. 3** that those chances have been oscillating over the years during 1982 through 1990. The probability for an existence of "quickening" attitude to report a case is $\hat{\phi}_{mle}$ if the case was reported in the same quarter of its occurrence. The chance for reporting an AIDS case in the same quarter of its occurrence is $1-\hat{\theta}_{mle,\hat{\phi}_{mle}}$ in the presence of "quickening odds" to report. The impact, $a_{\phi,\theta}$ of "quickening odds" is displayed in **Table 2**. Their values suggest that the "quickening odds" changed over the years during 1982-1990. The impact, $a_{mean,\phi,\theta}$ of "quickening odds" on mean has been reducing over the years during 1982 through 1990 (**Table 2**). Likewise, the $a_{dispersion,\phi,\theta}$ captures the impact of "quickening odds" to report on dispersion (**Table 2**). According to their values, the impact has been *increasing* over the years during 1982 through 1990.

Table 1. Reported number of AIDS cases, Y in a quarter during 1989-1990 in USA

Yr, Quarte	Q0	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q1 1	Q12	Q13	Q14	Q15	Q16+	$\bar{y}$	S_y
1982, Q1	31	49	32	10	5	10	5	2	4	0	2	0	0	0	0	0	35	10.88	15.56
1982, Q2	40	67	11	5	10	9	7	3	0	2	1	3	1	0	1	0	41	11.82	19.03
1982, Q3	78	73	32	21	12	11	1	3	1	2	2	1	1	1	2	2	50	17.24	25.77
1982, Q4	96	129	30	33	17	5	2	3	1	0	2	3	1	0	0	1	58	22.41	37.93
1983, Q1	134	177	68	34	14	12	4	7	3	4	3	4	2	1	2	0	67	31.53	51.91
1983, Q2	57	378	85	43	20	18	12	9	5	6	5	0	5	5	2	3	52	41.47	90.06
1983, Q3	69	420	113	34	19	12	10	10	4	4	3	4	3	3	7	4	50	45.24	101.10
1983, Q4	26	513	109	55	25	17	7	8	4	3	7	9	8	7	5	0	48	50.06	122.40
1984, Q1	55	675	151	59	32	26	18	8	9	7	7	4	9	7	5	6	70	67.53	161.00
1984, Q2	82	790	164	85	57	36	16	4	11	9	6	12	9	11	11	11	65	81.12	187.60
1984, Q3	108	845	241	112	47	40	18	16	15	9	8	8	5	13	11	7	70	92.53	203.20
1984, Q4	118	960	247	112	65	30	27	15	11	18	15	13	13	16	29	15	60	103.80	228.90
1985, Q1	146	1191	252	129	83	67	34	20	18	22	10	18	22	27	23	21	68	126.50	281.60
1985, Q2	160	1454	292	143	93	58	48	35	24	20	29	46	33	31	22	27	62	151.60	342.90
1985, Q3	152	1620	400	225	101	71	53	39	20	56	55	44	29	35	29	21	54	176.70	384.20
1985, Q4	97	1739	422	164	120	58	52	52	57	65	83	41	37	27	26	17	47	182.60	412.00
1986, Q1	148	2046	406	218	107	118	56	7	135	102	81	49	53	40	27	30	53	222.10	478.80
1986, Q2	562	2039	555	200	143	91	152	160	133	94	77	66	41	31	39	38	54	263.20	485.20
1986, Q3	232	2444	532	275	148	196	229	165	123	80	62	58	36	42	39	31		293.30	587.90
1986, Q4	181	2441	763	290	240	282	183	143	101	82	67	35	38	50				349.70	630.20
1987, Q1	224	2981	673	408	370	353	224	185	99	128	85	85	78	56				424.90	755.70
1987, Q2	129	3260	897	592	426	272	156	125	138	121	91	96	74					493.40	866.00
1987, Q3	96	3567	1207	569	374	227	195	138	102	118	117	85						563.00	999.50
1987, Q4	135	3847	1218	444	315	247	196	128	149	140	102							629.10	1114.00
1988, Q1	163	4401	1096	462	354	334	225	186	203	165								756.00	1311.00
1988, Q2	307	4608	968	500	372	284	222	231	182									853.00	1428.00
1988, Q3	332	4521	1186	569	334	317	268	193										959.30	1474.00
1988, Q4	256	4525	1327	487	375	387												1089.00	1560.00
1989, Q1	311	5016	1248	569	527	438												1352.00	1825.00
1989, Q2	342	5186	1370	814	512													1645.00	2018.00
1989, Q3	349	5124	1515	830														1955.00	2166.00
1989, Q4	192	4989	1745															2312.00	2453.00
1989, Q33	276	5646																2961.00	3797.00

Table 2. Parameter estimates with and without "quickening odds", their impacts on mean, dispersion, system's busy level, p-value for H_o: $\phi = 0$ and power of accepting H_1: $\phi^* = 0.5$

Yr,Quarter	$\hat{\phi}_{mel}$	$\hat{\phi}_{mel,\hat{\phi}mle}$	$\hat{\theta}_{mel,\hat{\phi}mle=0}$	$a_{\phi,\theta}$	$a_{mean,\phi,\theta}$	$a_{variance,\phi,\theta}$	$a_{busy,\phi,\theta}$ (1)	P (R>0.5)	pValue	Power	$odds_{\phi}$	$odds_{\phi,\theta}$
1982, Q1	0.912	0.151	0.916	2.265	0.403	0.838	1. 8492	0.554256	5.20E-05	4.00E-04	10.35	24.65
1982, Q2	0.947	0.117	0.922	2.215	0.294	0.874	1.9113	0.564873	7.60E-07	0.003	17.81	26.18
1982, Q3	0.953	0.094	0.945	2.161	0.224	0.901	1.9315	0.562217	1.90E-11	0.009	20.29	37.24
1982, Q4	0.976	0.063	0.957	2.111	0.142	0.935	1.9670	0.573760	4.70E-19	0.026	40.77	47.32
1983, Q1	0.981	0.047	0.969	2.081	0.103	0.951	1.9765	0.598687	9.00E-31	0.043	52.94	65.62
1983, Q2	0.994	0.029	0.976	2.054	0.062	0.970	1.9917	0.601831	8.30E-50	0.071	153.10	85.19
1983, Q3	0.996	0.026	0.978	2.049	0.062	0.973	1.9930	0.611652	2.20E-56	0.078	179.90	92.70
1983, Q4	0.997	0.023	0.980	2.044	0.048	0.976	1.9949	0.609265	1.60E-65	0.086	248.30	102.30
1984, Q1	0.997	0.017	0.985	2.033	0.036	0.982	1.9962	0.606182	2.80E-97	0.103	315.40	137.30
1984, Q2	0.997	0.015	0.988	2.027	0.030	0.985	1.9967	0.601002	4.00E-123	0.114	351.60	164.50
1984, Q3	0.998	0.013	0.989	2.024	0.027	0.987	1.9968	0.601526	7.00E-145	0.120	352.90	187.30
1984, Q4	0.998	0.012	0.990	2.022	0.024	0.988	1.9972	0.602589	5.00E-168	0.127	400.00	209.80
1985, Q1	0.998	0.010	0.992	2.018	0.020	0.990	1.9978	0.604369	0	0.138	499.20	255.30
1985, Q2	0.998	0.008	0.993	2.015	0.016	0.992	1.9983	0.600440	0	0.149	623.10	305.40
1985, Q3	0.999	0.007	0.994	2.013	0.014	0.993	1.9984	0.604213	0	0.156	657.60	355.70
1985, Q4	0.999	0.007	0.995	2.012	0.014	0.993	1.9986	0.599696	0	0.159	746.20	367.40
1986, Q1	0.998	0.006	0.996	2.010	0.011	0.994	1.9984	0.585430	0	0.168	809.00	446.50
1986, Q2	0.999	0.005	0.996	2.010	0.011	0.995	1.9988	0.598604	0	0.171	630.00	528.80
1986, Q3	0.999	0.005	0.997	2.008	0.009	0.996	1.9987	0.601272	0	0.179	884.30	588.80
1986, Q4	0.999	0.004	0.997	2.007	0.008	0.997	1.9989	0.600018	0	0.183	784.90	701.90
1987, Q1	0.999	0.003	0.998	2.006	0.007	0.997	1.9990	0.606213	0	0.192	918.00	852.30
1987, Q2	0.999	0.003	0.998	2.005	0.006	0.997	1.9992	0.616154	0	0.198	1026.00	1128.00
1987, Q3	0.999	0.003	0.998	2.004	0.005	0.998	1.9992	0.626100	0	0.205	1211.00	1261.00
1987, Q4	0.999	0.002	0.998	2.004	0.005	0.998	1.9993	0.635514	0	0.209	1343.00	1514.00
1988, Q1	0.999	0.002	0.999	2.003	0.004	0.998	1.9993	0.644970	0	0.216	1517.00	1709.00
1988, Q2	0.999	0.002	0.999	2.003	0.004	0.998	1.9992	0.649482	0	0.218	1538.00	1921.00
1988, Q3	0.999	0.002	0.999	2.003	0.004	0.998	1.9991	0.658648	0	0.218	1306.00	2182.00
1988, Q4	0.999	0.002	0.999	2.003	0.004	0.998	1.9991	0.673566	0	0.217	1143.00	2706.00
1989, Q1	0.999	0.002	0.999	2.002	0.003	0.998	1.9988	0.688098	0	0.220	1111.00	3294.00
1989, Q2	0.999	0.002	0.999	2.002	0.004	0.998	1.9977	0.710211	0	0.213	829.60	3915.00
1989, Q3	0.998	0.003	0.999	2.003	0.006	0.997	1.9965	0.760250	0	0.193	446.00	4633.00
1989, Q4	0.997	0.004	1.000	2.004	0.008	0.996	1.9995	0.900124	0	0.174	289.50	5926.00
1990, Q1	0.999	9E-04	1.000	2.001	0.002	0.999	1.9995	0.900124	0	0.235	1907.00	5926.00

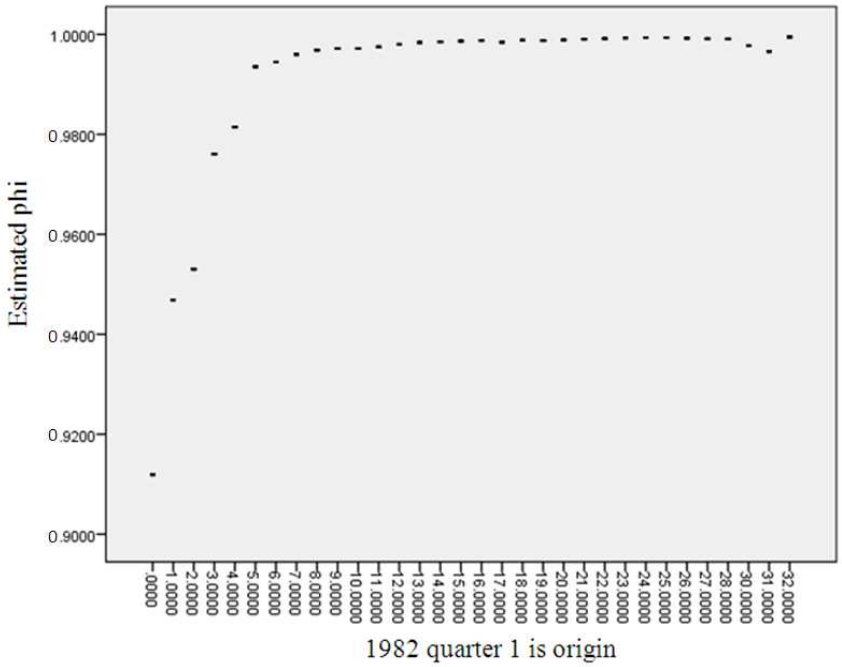

Fig. 1. Chance of "quickening" to report a delayed AIDS case

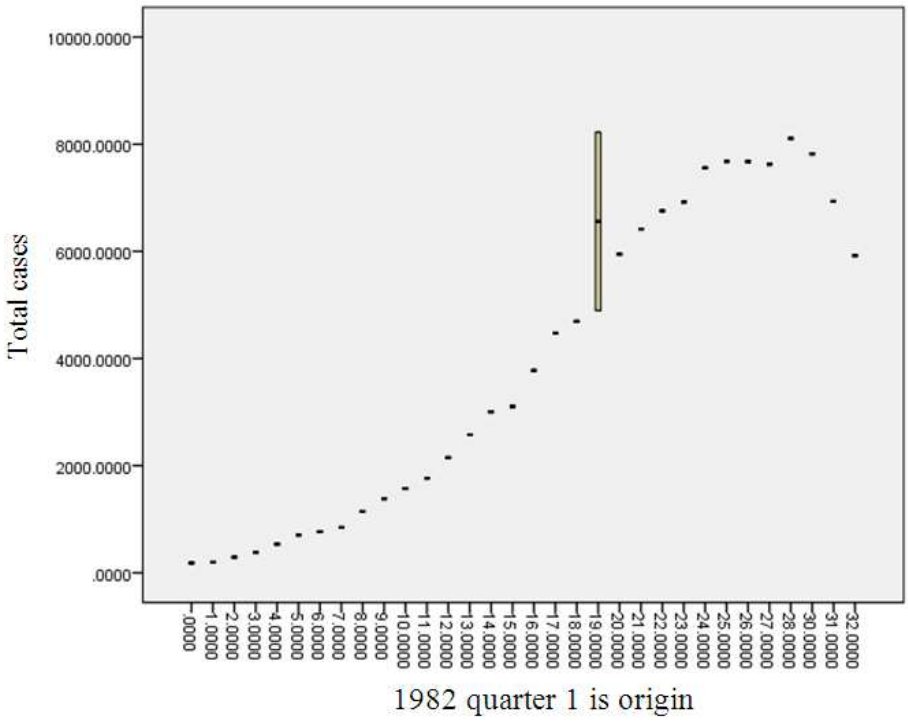

Fig. 2. The total number of reported AIDS cases over the years

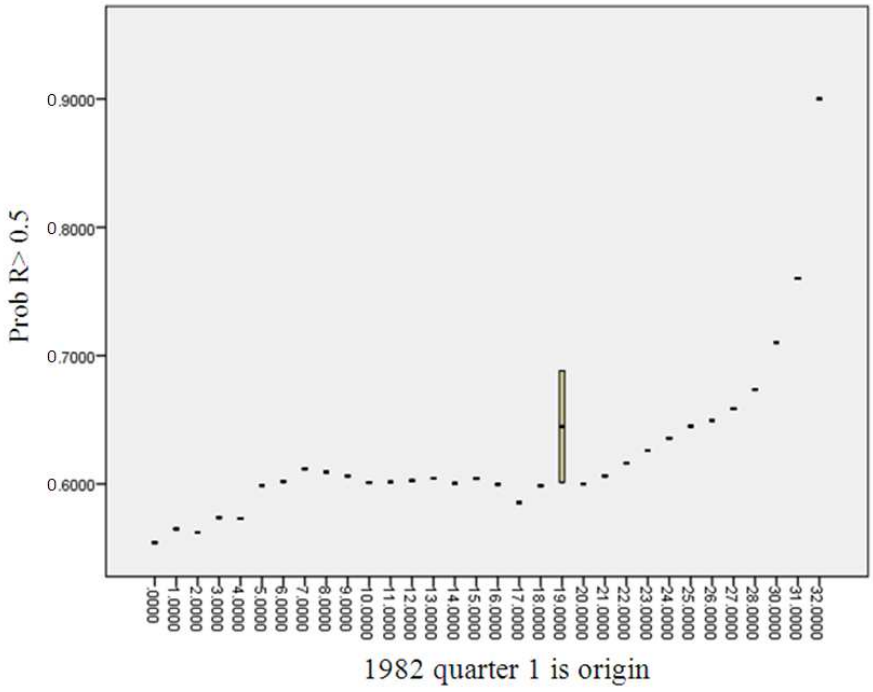

Fig. 3. Chance to report majority of the AIDS cases in the quarter of its occurrence

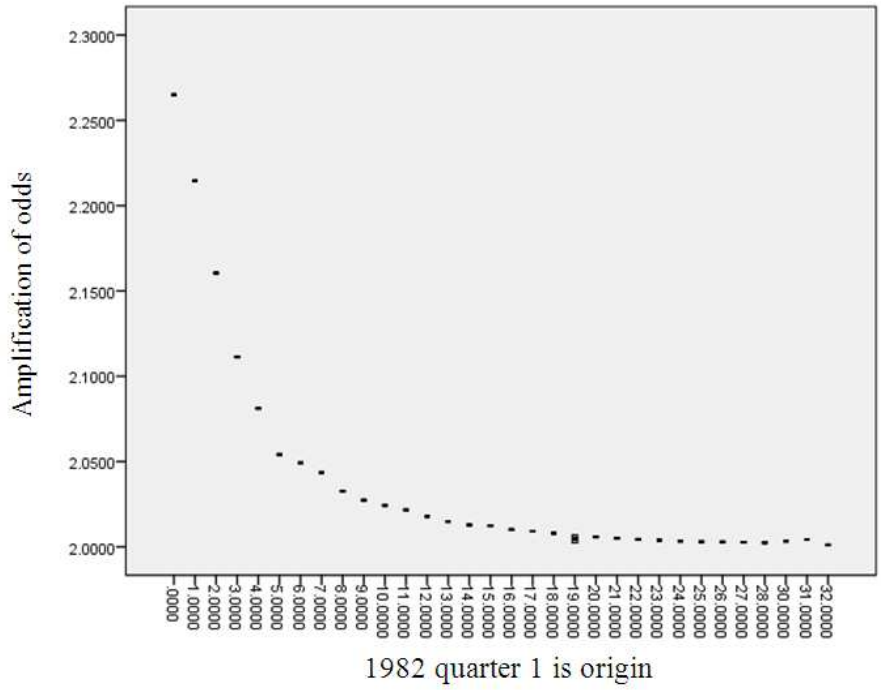

Fig. 4. Impact of "quickening odds" to report a delayed AIDS case

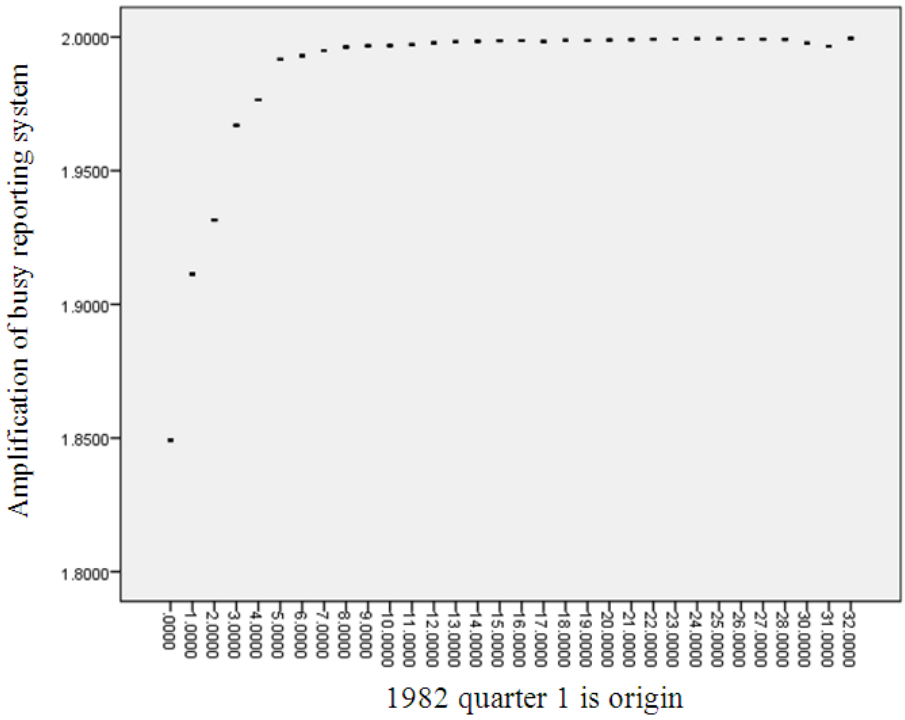

Fig. 5. Impact of "quickening odds" to report an AIDS case on system's busy level

Table 3. Reporting system's memory with "quickening odds "to report delayed AIDS cases

Yr,Quarter (m,t)=0,16	1,16	2,16	3,16	4,16	5,16	6,16	7,16	8,16	9,16	10,16	11,16	12,16	13,16	14,16	15,16
1982, Q1 1	0.984	0.969	0.956	0.943	0.931	0.920	0.910	0.900	0.891	0.882	0.874	0.8664	0.859	0.852	0.846
1982, Q2 1	0.986	0.972	0.960	0.948	0.937	0.927	0.917	0.908	0.899	0.891	0.884	0.8763	0.869	0.863	0.857
1982, Q3 1	0.992	0.984	0.977	0.970	0.963	0.957	0.950	0.945	0.939	0.934	0.929	0.9236	0.919	0.914	0.910
1982, Q4 1	0.995	0.989	0.984	0.979	0.975	0.970	0.966	0.962	0.958	0.954	0.950	0.9465	0.943	0.940	0.936
1983, Q1 1	0.997	0.994	0.991	0.988	0.985	0.983	0.980	0.978	0.975	0.973	0.970	0.9682	0.966	0.964	0.962
1983, Q2 1	0.998	0.996	0.994	0.993	0.991	0.989	0.987	0.986	0.984	0.983	0.981	0.9795	0.978	0.977	0.975
1983, Q3 1	0.998	0.997	0.995	0.994	0.992	0.991	0.989	0.988	0.986	0.985	0.984	0.9822	0.981	0.980	0.978
1983, Q4 1	0.999	0.997	0.996	0.995	0.993	0.992	0.991	0.990	0.988	0.987	0.986	0.9850	0.984	0.983	0.982
1984, Q1 1	0.999	0.998	0.998	0.997	0.996	0.995	0.995	0.994	0.993	0.993	0.992	0.9911	0.990	0.990	0.989
1984, Q2 1	0.999	0.999	0.998	0.998	0.997	0.997	0.996	0.996	0.995	0.995	0.994	0.9936	0.993	0.993	0.992
1984, Q3 1	1.000	0.999	0.999	0.998	0.998	0.997	0.997	0.997	0.996	0.996	0.995	0.9950	0.995	0.994	0.994
1984, Q4 1	1.000	0.999	0.999	0.999	0.998	0.998	0.998	0.997	0.997	0.997	0.996	0.9959	0.996	0.995	0.995
1985, Q1 1	1.000	1.000	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.997	0.9972	0.997	0.997	0.997
1985, Q2 1	1.000	1.000	0.999	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998	0.9980	0.998	0.998	0.998
1985, Q3 1	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.9985	0.998	0.998	0.998
1985, Q4 1	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.9986	0.998	0.998	0.998
1986, Q1 1	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999	0.999	0.9990	0.999	0.999	0.999
1986, Q2 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.9993	0.999	0.999	0.999
1986, Q3 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.9995	0.999	0.999	
\1986, Q4 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.9997			
1987, Q1 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.9998			
1987, Q2 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000					
1987, Q3 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000					
1987, Q4 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000						
1988, Q1 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000							
1988, Q2 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000								
1988, Q3 1	1.000	1.000	1.000	1.000	1.000	1.000									
1988, Q4 1	1.000	1.000	1.000	1.000	1.000										
1989, Q1 1	1.000	1.000	1.000	1.000											
1989, Q2 1	1.000	1.000	1.000												
1989, Q3 1	1.000	1.000													
1989, Q4 1	1.000														
1990, Q1 1															

The $a_{busy,\phi,\theta}$ captures the impact of "quickening odds" to report on system's busy level (**Table 2**). According to their values, the impact has been increasing from 1.84 to 1.99 over the years during 1982 through 1990. With notations Y and T-Y denoting respectively the number of reported, non-reported cases with m = 0.5 indicating the reported cases is 50% more than the non-reported cases in the same quarter of its occurrence, the chance $\Pr[\frac{Y}{T-Y} = R > m]$ is displayed in **Table 2**. Interestingly, their values suggest they had been more than 55% but oscillated over the years during 1982 through 1990. The p-value in **Table 2** indicates the chance for rejecting the true null hypothesis H_o: $\phi = 0$ and it confirms the existence of a significant "odds of quickening" to report. The power in **Table 2** implies the chance of accepting a true alternative hypothesis H_1: $\phi^* = 0.5$ with the level of significance $\alpha = 0.05$. Its oscillation hints the existence of varying administrative efforts to quickly report already delayed cases in the reporting medical system. It is worth examining how the reporting system's memory had been.

In a system with the absence of "quickening odds" to report an AIDS case, the system is recognized to follow a geometric probability distribution with no memory and the system's memory level $m_{\phi = 0,\theta}$ is just one. With an existence of "quickening odds" to report already delayed AIDS cases, the reporting system possesses a finite memory. The **Table 3** displays its memory level for the period 1982 through 1990.

2. CONCLUSION

This methodology is applicable to any delayed reporting system in other disciplines. In engineering, sports, e-marketing, healthcare insurance, stock-marketing, economic outcomes, cyber-crimes reporting with delay and the existence of efforts to quicken the reporting of already delayed cases are common. The contents of this article would help to discover non trivial impacts in those disciplines. Of course, many covariates are likely to influence the level of quickening efforts. A regression methodology is necessary to address the relevance of the covariates in a given investigation and it

could be developed with the significance of an estimated effort to quicken. A regression like the one in Pagano *et al*. (1994) is worthwhile and it is currently pursued for publication later.

3. REFERENCES

Bacchetti, P., 1996. Reporting delays of deaths with AIDS in the united states. J. Acquire Immune Defic. Syndr. Hum. Retovirol., 13: 363-367.

CDC, 1985. Revision of the case definition of acquired immunodeficiency syndrome for national reporting--united states. Morb. Mortal. Weekly Rep., 34: 373-5. PMID: 2989677

Chamberland, M.E., J.R. Allen, J.M. Monroe, N. Garcia and C. Morgan, 1985. Acquired immunodeficiency syndrome in New York city. Evaluation of an active surveillance system. J. Am. Med. Assoc., 254: 383-387. PMID: 4009865

Clegg, L.X., E.J. Feuer, M.P. Fay and B.F. Hankey, 2002. Impact of reporting delay and reporting error on cancer incidence rates and trends. J. National Cancer Inst., 94: 1537-45. DOI: 10.1093/jnci/94.20.1537

Cui, J., 1999. Nonparametric estimation of a delay distribution based on left-censored and right-truncated data. Biometrics, 55: 345-349. DOI: 10.1111/j.0006-341X.1999.00345.x

DeGruttola, V., X.M. Tu and M. Pagano, 1992. Pediatric AIDS in New York City: Estimating the distributions of infection, latency and reporting delay and projecting future incidence. J. Am. Stat. Assoc., 87: 633-640.

Fan, H.Y., R.F. Conner and L.P. Villarreal, 2004. AIDS: Science and Society. 4th Edn., Jones and Bartlett Learning, Boston, MA., ISBN-10: 076370086X, pp: 229.

Gebhardt, M.D., B.E. Neuenschwander and M. Zwahlen, 1998. Adjusting AIDS incidence for non-stationary reporting delays: A necessity for country comparisons. Eur. J. Epidemiol., 14: 595-503. DOI: 10.1023/A:1007406606892

Harris, J.E., 1990. Reporting delays and the incidence of AIDS. J. Am. Stat. Assoc., 85: 915-924.

Hay, J.W. and F.A. Wolak, 1994. A procedure for estimating the unconditional cumulative incidence curve and its variability for the human immunodeficiency virus. J. Royal Stat. Soc., C43: 599-624.

Lawless, J.F., 1994. Adjustments for reporting delays and the prediction of occurred but not reported events. Canadian J. Stat., 22: 15-31. DOI: 10.2307/3315826.n1

Lindsey, J.K., 1996. Fitting bivariate intensity functions, with an application to modelling delays in reporting acquired immune deficiency syndrome. J. Royal Stat. Soc., 159: 125-131.

MacArthur, C., L. Pendleton and A. Smith, 1985. Treatment delay in patients with bladder tumours. J. Epidemiol. Community Health, 39: 63-66. DOI: 10.1136/jech.39.1.63

Midthune, D.N., M.P. Fay, L.X. Clegg and E.J. Feuer, 2005. Modeling reporting delays and reporting corrections in cancer registry data. J. Am. Stat. Assoc., 100: 61-70. DOI: 10.1198/016214504000001899

Pagano, M., X.M. Tu, V.D. Gruttola and S. MaWhinney, 1994. Regression analysis of censored and truncated data: Estimating reporting-delay distributions and AIDS incidence from surveillance data. Biometrics, 50: 1203-1214. PMID: 7787003

Stuart, A. And K. Ord, 1994. Kendall's Advanced Theory of Statistics, Distribution Theory. 1st Edn., Wiley, Chichester, ISBN-10: 0470665300, pp: 700.

Tabnak, F., H.G. Müller, J.L. Wang, J.M. Chiou and R. Sun, 2000. A change-point model for reporting delays under change of AIDS case definition. Eur. J. Epidemiol., 16: 1135-1141. DOI: 10.1023/A:1010955827954

Wald, A., 1943. Tests of statistical hypotheses concerning several parameters when the number of observations is large. Trans. Am. Math. Soc., 54: 426-482.

Zou, J., L. Huang, D. Midthune, M.J. Horner and M. Krapcho *et al*., 2009. Effect of reporting year on delay modeling. Stat. Res. Applic. Branch, NCI.