Skeleton Primitive Extraction Method on Textures with Different Nonlinear Wavelets

[1]V. Vijaya Kumar, [2]U.S.N. Raju, [3]P. Premchand and [4]A. Suresh
[1,2]Department of Computer Science Engineering and Information Technology,
GIET, Rajahmundry, A.P., India
[3]Department of Computer Science Engineering, Osmania University, Hyderabad, A.P., India
[4]Indian Railway Service and JNT University Research Scholar, A.P., India

Abstract: Problem Statement: A novel method for dominant skeleton extraction of textures using different nonlinear wavelet transforms is proposed in this study. In the present study 3×3 masks are used for extraction of skeleton primitives. For a 3×3 skeleton primitive there will be 2^9 skeleton primitive combinations. But the present study considered a skeleton primitive for the skeletonization purpose if and only if its center pixel is one and skeleton primitives are represented by the corresponding skeleton primitive weight. By this, there will be 2^8 combinations of skeleton primitives. **Approach**: The skeleton primitive are used for evaluating skeleton points. The skeleton of an object has the property that it is reduced to one point when the skeleton primitive used for the skeletonization is exactly homothetic to the object. The dominant skeleton subset is evaluated by counting the skeleton points. The skeleton subset that leads to the least skeleton points will be the resultant skeleton subset. The present study classified the textures based on two methods. In the first method textures are classified based on skeleton primitive weight, which is nothing but based on skeleton primitive combination. In the second method classification is made based on distance function of skeleton points. **Results**: The proposed method is applied on 24 Brodatz textures using the three nonlinear wavelet transformed textures. By this the dominant skeleton primitive weight is obtained for each texture. Based on the number of skeleton points distance measures are calculated based on which texture classification is obtained. **Conclusions**: The two methods are applied on Brodatz textures using different nonlinear wavelet transforms which classified the textures. The first method is appropriate if one need to classify based on skeleton subsets. The second method is appropriate if the classification is to be done based on least number of skeleton points.

Keywords: Dominant skeleton subset, combinations of skeleton primitives, homothetic, skeleton points, nonlinear wavelets

INTRODUCTION

Texture is the term used to characterize the surface of a given phenomenon in an image and it is undoubtedly one of the principal features used in image processing. The different texture analysis methods are structural, statistical, model based and transform based methods. Structural approaches represent texture by well defined primitives (microtexture) and a hierarchy of spatial arrangements (macrotexture) of those primitives. Statistical approaches represent the texture indirectly by the non-deterministic properties that govern the distributions and relationships between the grey levels of an image. Model based texture analysis using fractal and stochastic models, attempt to interpret an image texture by use of generative image model and stochastic model respectively. Transform methods of texture analysis, such as Fourier and wavelet transforms[1-3] represent an image in a space whose co-ordinate system has an interpretation that is closely related to the characteristics of a texture. Texture classification plays an important role in the interpretation and analysis of terrain images, as well as radiographic and microscopic images. Method of texture classification can be broadly divided into two major categories. The first is texture segmentation, based on features with a high degree of spatial localization e.g. with some form of edge detection. The second approach to texture classification is based on a discrimination function using several texture characteristics. Scientists and practitioners have made great efforts in developing advance classification

Corresponding Author: V. Vijaya Kumar, Department of Computer Science Engineering and Information Technology, GIET, Rajahmundry, A.P., India

approaches and techniques for improving classification accuracy[4-6].

Study of patterns on textures is recognized as an important step in characterization and classification of textures. Textures are classified recently by various pattern methods: preprocessed images[13], long linear patterns[8,12] and edge direction movements[7], Avoiding Complex Patterns[11], marble texture description[10] etc. Textures are also described and classified by using various wavelet transforms: One based on primitive patterns[14] and another based on statistical parameters[9].

Skeletonization is a global space domain technique for shape representation[15]. It has been studied extensively since skeletons have attractive properties which make them suitable for structural pattern recognition[15,16]. There are two types of skeletonization methods: Pixel-based and non-pixel-based. In a pixel-based method, all pixels inside a shape are used in the skeletonization process. Pixel based methods often use thinning techniques[16,17] or distance transforms[17,18]. In a non-pixel-based method, only the contour pixels of a shape are used for skeletonization. The skeleton of the shape is analytically derived from its contour[19,20]. Skeletonization has been a part of image processing for a wide variety of applications[21]. Digital skeletons, generated by thinning algorithms, are often used to represent objects in a binary digital image for shape analysis and classification. Thinning is a process of reducing patterns to their skeleton; however, skeleton is defined as a set of thin lines, arcs and curves (usually one pixel thick), which are connected with each other in such a way that the geometrical and topological properties of its originating object must be preserved.

Wavelet analysis is known as a *linear* tool. However, it is to be recognized that nonlinear extensions are possible[23-25]. The *lifting scheme*, recently introduced by Sweldens[22,26,27] for a predecessor of this scheme, known as a ladder network, has provided a useful tool for constructing nonlinear wavelet transforms. The enormous flexibility and freedom that the lifting scheme offers has challenged researchers to develop various nonlinear wavelet transforms[23-25]. Here we present a family of nonlinear wavelets based on morphological operators. The simplest nontrivial example of a morphological wavelet is the so called morphological Haar wavelet. The morphological haar wavelets are of two types: Morphological Haar Erosion Wavelet (MHEW) and Morphological Haar Dilation Wavelet (MHDW). The other nonlinear wavelet is Morphological Binary Wavelet transform (MBW).

MATERIALS AND METHODS

On a 3×3 mask by assuming center pixel as one always, one can have 2^8 combinations. However the present study has not considered the following skeleton primitive as shown in Fig. 1 for extracting skeleton primitives. By this there will be a total of 255 skeleton primitives on a 3×3 mask. These skeleton primitives are used for evaluating skeleton points. The skeleton primitives are represented by weight based system as shown in Fig. 2 which are called as skeleton primitive weight (SPW). Some of the skeleton primitive elements with their weights are represented in the Fig. 3.

0	0	0
0	1	0
0	0	0

Fig. 1: Skeleton primitive that is not been considered for skeletonization

$p\times2^0$	$p\times2^1$	$p\times2^2$
$p\times2^4$		$p\times23$
$p\times2^5$	$p\times2^6$	$p\times2^7$

Fig. 2: Skeleton primitive weight representation

1	1	0
0	1	0
0	0	0

(a)

1	0	1
0	1	0
0	0	0

(b)

1	1	1
0	1	0
0	0	0

(c)

1	1	1
1	1	0
0	0	0

(d)

1	1	1
1	1	0
0	0	0

(e)

1	1	1
1	1	1
1	0	0

(f)

1	0	1
0	1	0
1	0	1

(g)

1	1	1
0	1	1
0	1	1

(h)

1	1	1
1	1	1
1	1	1

(i)

Fig. 3: Representation of Skeleton primitives with corresponding weights (a) 3 (b) 5 (c) 7 (d) 23 (e) 31 (f) 63 (g) 165 (h) 207 (i)255

We propose in this study a novel method of the texture primitive description that requires no assumption on the distribution of grain sizes or the granulometric moments of the primitives. We employ the morphological skeleton for this method. The most commonly employed morphological skeleton of a binary object is explained intuitively as follows: At first

we locate the largest magnification included within the object and cover the object by sweeping the magnification within the object. Then gradually smaller magnifications are employed for covering the residual area until the whole object is covered. The skeleton varies depending on the shape of the skeleton primitive. If the skeleton primitive is homothetic to the object, the object is covered with only one magnification of the skeleton primitive. In this case the skeleton is reduced to one point. We consider here obtaining the skeleton from a binary texture. It is derived from the above property that the total number of pixels within the skeleton is the minimum when the skeleton primitive is homothetic to the primitive, if we assume that the texture is composed of one primitive, i.e. contains grains that are magnifications of the primitive. This indicates that the primitive is described by the optimal skeleton primitive minimizing the total number of pixels within the skeleton. This primitive description method has an advantage that no assumption on the sizing distribution of grains in the texture is required.

Skeletonization is done according to Algorithm 1. The proposed texture skeleton primitive extraction method on nonlinear wavelets is given in the form of flowchart in Fig. 4. The three nonlinear wavelet transforms used in the present study are MHEW, MHDW and MBW.

Algorithm 1: To find Skeleton of an image
Begin
1. Let $k=1$ and A be the image.
2. $E(A,kB)$, Erode the image k number of times with skeleton primitive B.
3. $O(A,B)$ Open the image with skeleton primitive B.
4. Find the kth Skeleton Subset $S_k=E(A,kB)-O(E(A,kB)$.
5. $k=k+1$
6. $T(A)=E(A,kB)$.
7. If $(T(A)\neq\varphi)$ goto 4.
8. $SK(A,B)$ is the union of all the Skeleton Subsets $S_k(A)$.

End

RESULTS

Evaluation: The proposed Texture Skeleton Primitive extraction method on nonlinear wavelets is applied on 24 Brodatz textures[28] shown in Fig. 5, using 255 skeleton primitive combinations. The process of applying all skeleton primitive combinations for skeletonization is tedious; however it gives accurate

result i.e. the exact combination of skeleton subset. The Table 1, 2 and 3 indicates the dominant skeleton primitive and the corresponding *SPW* for all 24 Brodatz textures using MHEW, MHDW and MBW wavelet transform respectively.
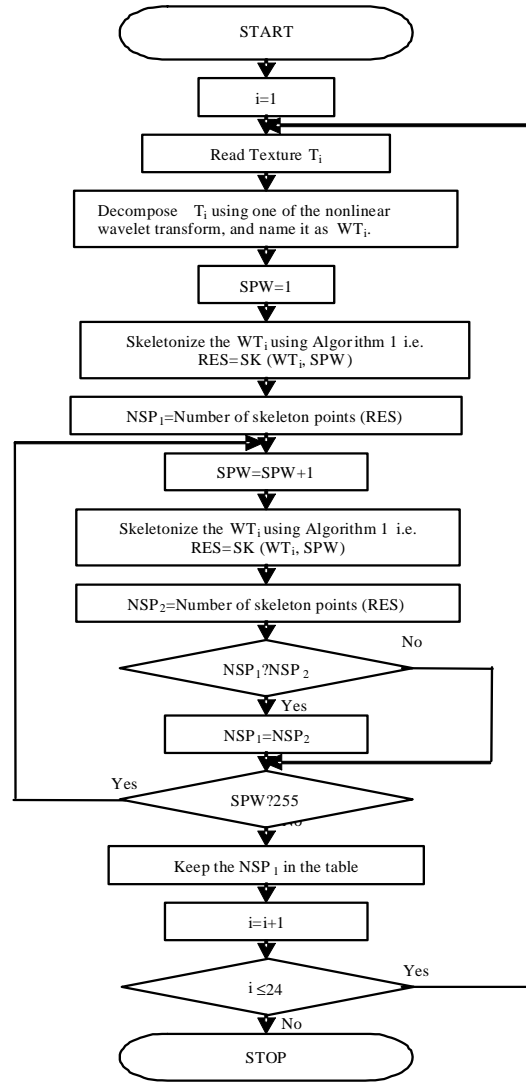
Fig. 4: Flow chart for texture skeleton primitive extraction using nonlinear wavelets.

Based on these results, the textures are classified. From the number of skeleton points, the distance measures are calculated and shown in Table 4, 5 and 6 for the MHE, MHD and MB wavelets respectively. The textures are classified based on these distance measures also. The classification results with lag values 30 and 50 are calculated and are shown in Tables 7 and 8.

DISCUSSION

From Table 1, 2 and 3 it is clearly evident that a total of 17, 20 and 18 textures are showing a dominant skeleton primitive with a weight of 255 for MHE, MHD and MB wavelet transforms respectively as show below.
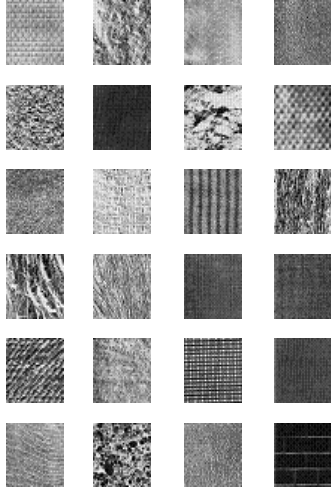

Fig. 5: Twenty four textures from Brodatz album. Row 1: D_1, D_2, D_3, D_4. Row 2: D_5, D_6, D_7, D_8. Row 3: D_9, D_{10}, D_{11}, D_{12}. Row 4: D_{13}, D_{15}, D_{16}, D_{17}. Row 5: D_{18}, D_{19}, D_{20}, D_{21}. Row 6:D_{22}, D_{23}, D_{24}, D_{25}.

Table 1: Textures with corresponding skeleton primitive weights by using MHEWT

Texture	No. of skeleton points	Dominant skeleton primitive weight
D_1	3940	255
D_2	4265	255
D_3	6881	19
D_4	7155	223
D_5	4428	255
D_6	3904	24
D_7	3717	255
D_8	2222	255
D_9	6182	255
D_{10}	5600	255
D_{11}	6379	255
D_{12}	4532	255
D_{13}	3850	255
D_{15}	6020	255
D_{16}	7648	255
D_{17}	7550	139
D_{18}	3822	255
D_{19}	5321	255
D_{20}	2688	255
D_{21}	2974	24
D_{22}	5384	206
D_{23}	3429	255
D_{24}	6989	66
D_{25}	3980	255

Table 2: Textures with corresponding skeleton primitive weights by using MHDWT

Texture	No. of skeleton points	Dominant skeleton primitive weight
D_1	5463	255
D_2	4684	255
D_3	8201	255
D_4	7698	251
D_5	4772	255
D_6	5469	153
D_7	3111	255
D_8	2167	255
D_9	7216	255
D_{10}	6371	255
D_{11}	6181	255
D_{12}	5715	255
D_{13}	4545	255
D_{15}	6868	255
D_{16}	6128	255
D_{17}	6898	54
D_{18}	4177	255
D_{19}	5763	255
D_{20}	4683	255
D_{21}	6873	112
D_{22}	7093	255
D_{23}	3524	255
D_{24}	8087	255
D_{25}	2171	255

Table 3: Textures with corresponding skeleton primitive weights by using MBWT

Texture	No. of skeleton points	Dominant skeleton primitive weight
D_1	4221	255
D_2	4542	255
D_3	8244	255
D_4	7591	223
D_5	4670	255
D_6	4452	153
D_7	3584	255
D_8	2150	255
D_9	6922	255
D_{10}	5811	255
D_{11}	6398	255
D_{12}	5460	255
D_{13}	4267	255
D_{15}	6988	255
D_{16}	5883	255
D_{17}	6746	109
D_{18}	4032	255
D_{19}	5717	255
D_{20}	3568	255
D_{21}	4125	24
D_{22}	6795	206
D_{23}	3512	255
D_{24}	7920	115
D_{25}	2687	255

$MHEWC_1 = \{D_1, D_2, D_5, D_7, D_8, D_9, D_{10}, D_{11}, D_{12}, D_{13}, D_{15}, D_{16}, D_{18}, D_{19}, D_{20}, D_{23}, D_{25}\}$

$MHDWC_1 = \{D_1, D_2, D_3, D_5, D_7, D_8, D_9, D_{10}, D_{11}, D_{12}, D_{13}, D_{15}, D_{16}, D_{18}, D_{19}, D_{20}, D_{22}, D_{23}, D_{24}, D_{25}\}$

$MBWC_1 = \{D_1, D_2, D_3, D5, D_7, D_8, D_9, D_{10}, D_{11}, D_{12}, D_{13}, D_{15}, D_{16}, D_{18}, D_{19}, D_{20}, D_{23}, D_{25}\}$

Out of these, 17 textures are having a dominant skeleton primitive with a weight of 255 for all three nonlinear wavelet transforms considered and they are listed below.

$\{D_1, D_2, D_5, D_7, D_8, D_9, D_{10}, D_{11}, D_{12}, D_{13}, D_{15}, D_{16}, D_{18}, D_{19}, D_{20}, D_{23}, D_{25}\}$

And no other texture is having any other skeleton primitive with common weight in all three nonlinear wavelet transforms: MHEW, MHDW and MBW. By this, the present study evaluates a common classification rate based on skeleton primitives as 70.83%. Further the present study classified textures based on their least number of skeleton points using distance function as follows. By distance function[29] we say that two textures are similar if and only if they contain the same number of primitive patterns or same percentage of occurrence of patterns as given in Eq. 1.

$$D(i) = \sum_{j=1}^{p} abs\left(F_j(x) - F_j(i)\right) = 0 \qquad (1)$$

where p stands for the total number of features used, i = 1 to Q (Q is the number of classes in the database), $f_j(x)$, represents the jth feature of unknown texture class (x) and $f_j(i)$ represents the jth feature of texture belonging to i^{th} class.

Equation 1 is a rare phenomenon for classification of textures based on skeleton points, because they appear in large number. To avoid this, a lag value based distance function is given in the Eq. 2.

$$D(i) = \sum_{j=1}^{p} abs\left(F_j(x) - F_j(i)\right) \leq l_i \qquad (2)$$

Equation 2 is modified in the present study as:

$$D(WT_i, WT_j) = \left| NSP_i - NSP_j \right| \leq l_i \qquad (3)$$

where WT_i and WT_j are the wavelet transformed Brodatz textures, $1 \leq i$, $j \leq 24$, NSP_i and NSP_j are the number of skeleton points in the Skeletons of the two textures. This Eq. 3 suggests that two textures are similar if their distances or percentage of occurrence is less than or equal to lag value 'l_i'. This indicates that lag value plays crucial role and by increasing lag value the percentage of correct classification (PCC) increases. The distances among all 24 textures based on number of skeleton points using MHE, MHD and MB wavelet transform are listed in Table 4, 5 and 6 respectively. Based on the lag value a classification is resulted in the following way. That is two or more textures belong to the same class C_i if they differ with the lag value less than or equal to l_i.

Table 4: Distance among the textures based on number of skeleton points using MHE Wavelet Transform

	D₁	D₂	D₃	D₄	D₅	D₆	D₇	D₈	D₉	D₁₀	D₁₁	D₁₂	D₁₃	D₁₅	D₁₆	D₁₇	D₁₈	D₁₉	D₂₀	D₂₁	D₂₂	D₂₃	D₂₄	D₂₅
D₁	0	18	54	57	22	6	15	41	47	41	49	24	9	46	61	60	11	37	35	31	38	23	55	6
D₂	18	0	51	54	13	19	23	45	44	37	46	16	20	42	58	57	21	32	40	36	33	29	52	17
D₃	54	51	0	17	50	55	56	68	26	36	22	48	55	29	28	26	55	39	65	63	39	59	10	54
D₄	57	54	17	0	52	57	59	70	31	39	28	51	57	34	22	20	58	43	67	65	42	61	13	56
D₅	22	13	50	52	0	23	27	47	42	34	44	10	24	40	57	56	25	30	42	38	31	32	51	21
D₆	6	19	55	57	23	0	14	41	48	41	50	25	7	46	61	60	9	38	35	30	38	22	56	9
D₇	15	23	56	59	27	14	0	39	50	43	52	29	12	48	63	62	10	40	32	27	41	17	57	16
D₈	41	45	68	70	47	41	39	0	63	58	64	48	40	62	74	73	40	56	22	27	56	35	69	42
D₉	47	44	26	31	42	48	50	63	0	24	14	41	48	13	38	37	49	29	59	57	28	52	28	47
D₁₀	41	37	36	39	34	41	43	58	24	0	28	33	42	20	45	44	42	17	54	51	15	47	37	40
D₁₁	49	46	22	28	44	50	52	64	14	28	0	43	50	19	36	34	51	33	61	58	32	54	25	49
D₁₂	24	16	48	51	10	25	29	48	41	33	43	0	26	39	56	55	27	28	43	39	29	33	50	23
D₁₃	9	20	55	57	24	7	12	40	48	42	50	26	0	47	62	61	5	38	34	30	39	21	56	11
D₁₅	46	42	29	34	40	46	48	62	13	20	19	39	47	0	40	39	47	26	58	55	25	51	31	45
D₁₆	61	58	28	22	57	61	63	74	38	45	36	56	62	40	0	10	62	48	70	68	48	65	26	61
D₁₇	60	57	26	20	56	60	62	73	37	44	34	55	61	39	10	0	61	47	70	68	47	64	24	60
D₁₈	11	21	55	58	25	9	10	40	49	42	51	27	5	47	62	61	0	39	34	29	40	20	56	13
D₁₉	37	32	39	43	30	38	40	56	29	17	33	28	38	26	48	47	39	0	51	48	8	43	41	37
D₂₀	35	40	65	67	42	35	32	22	59	54	61	43	34	58	70	70	34	51	0	17	52	27	66	36
D₂₁	31	36	63	65	38	30	27	27	57	51	58	39	30	55	68	68	29	48	17	0	49	21	63	32
D₂₂	38	33	39	42	31	38	41	56	28	15	32	29	39	25	48	47	40	8	52	49	0	44	40	37
D₂₃	23	29	59	61	32	22	17	35	52	47	54	33	21	51	65	64	20	43	27	21	44	0	60	23
D₂₄	55	52	10	13	51	56	57	69	28	37	25	50	56	31	26	24	56	41	66	63	40	60	0	55
D₂₅	6	17	54	56	21	9	16	42	47	40	49	23	11	45	61	60	13	37	36	32	37	23	55	0

Table 5: Distance among the textures based on number of skeleton points using MHD Wavelet Transform

	D₁	D₂	D₃	D₄	D₅	D₆	D₇	D₈	D₉	D₁₀	D₁₁	D₁₂	D₁₃	D₁₅	D₁₆	D₁₇	D₁₈	D₁₉	D₂₀	D₂₁	D₂₂	D₂₃	D₂₄	D₂₅
D₁	0	28	52	47	26	2	48	57	42	30	27	16	30	37	26	38	36	17	28	38	40	44	51	57
D₂	28	0	59	55	9	28	40	50	50	41	39	32	12	47	38	47	23	33	1	47	49	34	58	50
D₃	52	59	0	22	59	52	71	78	31	43	45	50	60	37	46	36	63	49	59	36	33	68	11	78
D₄	47	55	22	0	54	47	68	74	22	36	39	45	56	29	40	28	59	44	55	29	25	65	20	74
D₅	26	9	59	54	0	26	41	51	49	40	38	31	15	46	37	46	24	31	9	46	48	35	58	51
D₆	2	28	52	47	26	0	49	57	42	30	27	16	30	37	26	38	36	17	28	37	40	44	51	57
D₇	48	40	71	68	41	49	0	31	64	57	55	51	38	61	55	62	33	51	40	61	63	20	71	31
D₈	57	50	78	74	51	57	31	0	71	65	63	60	49	69	63	69	45	60	50	69	70	37	77	2
D₉	42	50	31	22	49	42	64	71	0	29	32	39	52	19	33	18	55	38	50	19	11	61	30	71
D₁₀	30	41	43	36	40	30	57	65	29	0	14	26	43	22	16	23	47	25	41	22	27	53	41	65
D₁₁	27	39	45	39	38	27	55	63	32	14	0	22	40	26	7	27	45	20	39	26	30	52	44	63
D₁₂	16	32	50	45	31	16	51	60	39	26	22	0	34	34	20	34	39	7	32	34	37	47	49	60
D₁₃	30	12	60	56	15	30	38	49	52	43	40	34	0	48	40	49	19	35	12	48	50	32	60	49
D₁₅	37	47	37	29	46	37	61	69	19	22	26	34	48	0	27	5	52	33	47	2	15	58	35	69
D₁₆	26	38	46	40	37	26	55	63	33	16	7	20	40	27	0	28	44	19	38	27	31	51	44	63
D₁₇	38	47	36	28	46	38	62	69	18	23	27	34	49	5	28	0	52	34	47	5	14	58	34	69
D₁₈	36	23	63	59	24	36	33	45	55	47	45	39	19	52	44	52	0	40	22	52	54	26	63	45
D₁₉	17	33	49	44	31	17	51	60	38	25	20	7	35	33	19	34	40	0	33	33	36	47	48	60
D₂₀	28	1	59	55	9	28	40	50	50	41	39	32	12	47	38	47	22	33	0	47	49	34	58	50
D₂₁	38	47	36	29	46	37	61	69	19	22	26	34	48	2	27	5	52	33	47	0	15	58	35	69
D₂₂	40	49	33	25	48	40	63	70	11	27	30	37	50	15	31	14	54	36	49	15	0	60	32	70
D₂₃	44	34	68	65	35	44	20	37	61	53	52	47	32	58	51	58	26	47	34	58	60	0	68	37
D₂₄	51	58	11	20	58	51	71	77	30	41	44	49	60	35	44	34	63	48	58	35	32	68	0	77
D₂₅	57	50	78	74	51	57	31	2	71	65	63	60	49	69	63	69	45	60	50	69	70	37	77	0

Table 6: Distance among the textures based on number of skeleton points using MB Wavelet Transform

	D₁	D₂	D₃	D₄	D₅	D₆	D₇	D₈	D₉	D₁₀	D₁₁	D₁₂	D₁₃	D₁₅	D₁₆	D₁₇	D₁₈	D₁₉	D₂₀	D₂₁	D₂₂	D₂₃	D₂₄	D₂₅
D₁	0	18	63	58	21	15	25	46	52	40	47	35	7	53	41	50	14	39	26	10	51	27	61	39
D₂	18	0	61	55	11	9	31	49	49	36	43	30	17	49	37	47	23	34	31	20	47	32	58	43
D₃	63	61	0	26	60	62	68	78	36	49	43	53	63	35	49	39	65	50	68	64	38	69	18	75
D₄	58	55	26	0	54	56	63	74	26	42	35	46	58	25	41	29	60	43	63	59	28	64	18	70
D₅	21	11	60	54	0	15	33	50	47	34	42	28	20	48	35	46	25	32	33	23	46	34	57	45
D₆	15	9	62	56	15	0	29	48	50	37	44	32	14	50	38	48	20	36	30	18	48	31	59	42
D₇	25	31	68	63	33	29	0	38	58	47	53	43	26	58	48	56	21	46	4	23	57	8	66	30
D₈	46	49	78	74	50	48	38	0	69	61	65	58	46	70	61	68	43	60	38	44	68	37	76	23
D₉	52	49	36	26	47	50	58	69	0	33	23	38	52	8	32	13	54	35	58	53	11	58	32	65
D₁₀	40	36	49	42	34	37	47	61	33	0	24	19	39	34	8	31	42	10	47	41	31	48	46	56
D₁₁	47	43	43	35	42	44	53	65	23	24	0	31	46	24	23	19	49	26	53	48	20	54	39	61
D₁₂	35	30	53	46	28	32	43	58	38	19	31	0	35	39	21	36	38	16	43	37	37	44	50	53
D₁₃	7	17	63	58	20	14	26	46	52	39	46	35	0	52	40	50	15	38	26	12	50	27	60	40
D₁₅	53	49	35	25	48	50	58	70	8	34	24	39	52	0	33	16	54	36	58	54	14	59	31	66
D₁₆	41	37	49	41	35	38	48	61	32	8	23	21	40	33	0	29	43	13	48	42	30	49	45	57
D₁₇	50	47	39	29	46	48	56	68	13	31	19	36	50	16	29	0	52	32	56	51	7	57	34	64
D₁₈	14	23	65	60	25	20	21	43	54	42	49	38	15	54	43	52	0	41	22	10	53	23	62	37
D₁₉	39	34	50	43	32	36	46	60	35	10	26	16	38	36	13	32	41	0	46	40	33	47	47	55
D₂₀	26	31	68	63	33	30	4	38	58	47	53	43	26	58	48	56	22	46	0	24	57	7	66	30
D₂₁	10	20	64	59	23	18	23	44	53	41	48	37	12	54	42	51	10	40	24	0	52	25	62	38
D₂₂	51	47	38	28	46	48	57	68	11	31	20	37	50	14	30	7	53	33	57	52	0	57	34	64
D₂₃	27	32	69	64	34	31	8	37	58	48	54	44	27	59	49	57	23	47	7	25	57	0	66	29
D₂₄	61	58	18	18	57	59	66	76	32	46	39	50	60	31	45	34	62	47	66	62	34	66	0	72
D₂₅	39	43	75	70	45	42	30	23	65	56	61	53	40	66	57	64	37	55	30	38	64	29	72	0

When lag value is 30 the classes resulted based on least number of skeleton primitive points for the three nonlinear wavelets are shown in Table 7. In the same way, when the lag value is 50 the resultant classes are shown in Table 8.

If we assume MB wavelet transform as standard one then the classification rates are evaluated for each class with other nonlinear wavelets and they are listed in Table 9 and Table 10 for lag value 30 and 50 respectively. The classification rates are evaluated by the following formula: Percentage of classification rate = Number of common textures of W_x with respect to W_y for class Ci/ Number of textures in W_y for class C_i that means if W_y has got 'l' textures in class C_i and W_x has got 'l+m' textures then by the above formula shows 100% of correct classification for class C_i. However this effects for another class because of extra 'm' textures. By this the above formula guarantees that overall texture classification rate does not change and comparison rate is not going to be effected.

Table 7: Classes of textures for the three nonlinear wavelets when the lag value is 30

Wavelet	Class	Textures
MHEWT	C_1	$\{D_1, D_2, D_5, D_6, D_7, D_{12}, D_{13}, D_{18}, D_{25}\}$
	C_2	$\{D_3, D_4, D_{11}, D_{24}\}$
	C_3	$\{ D_8, D_{20}, D_{21}\}$
	C_4	$\{ D_9, D_{10}, D_{15}, D_{19}, D_{22}\}$
	C_5	$\{ D_{16}, D_{17}\}$
	C_6	$\{ D_{23}\}$
MHDWT	C_1	$\{ D_1, D_2, D_5, D_6, D_{13}, D_{20}\}$
	C_2	$\{ D_3, D_4, D_{24}\}$
	C_3	$\{ D_7, D_{23}\}$
	C_4	$\{ D_8, D_{25}\}$
	C_5	$\{ D_9, D_{10}, D_{15}, D_{17}, D_{21}, D_{22}\}$
	C_6	$\{ D_{11}, D_{12}, D_{16}, D_{19}\}$
	C_7	$\{ D_{18}\}$
MBWT	C_1	$\{ D_1, D_2, D_5, D_6, D_{13}, D_{18}, D_{21}\}$
	C_2	$\{ D_3, D_4, D_{24}\}$
	C_3	$\{ D_7, D_{20}, D_{23}, D_{25}\}$
	C_4	$\{ D_8\}$
	C_5	$\{ D_9, D_{11}, D_{15}, D_{17}, D_{22}\}$
	C_6	$\{ D_{10}, D_{12}, D_{16}, D_{19}\}$

Table 8: Classes of textures for the three nonlinear wavelets when the lag value is 50

Wavelet	Class	Textures
MHEWT	C_1	$\{D_1,D_2,D_5,D_6,D_7,D_8,D_{12},D_{13},D_{18},D_{20},D_{21},D_{23},D_{25}\}$
	C_2	$\{D_3,D_4,D_9,D_{10},D_{11},D_{15},D_{16},D_{17},D_{19},D_{22},D_{24}\}$
MHDWT	C_1	$\{D_1,D_2,D_5,D_6,D_7,D_{13},D_{18},D_{20},D_{23}\}$
	C_2	$\{D_3,D_4,D_9,D_{10},D_{11},D_{12},D_{15},D_{16},D_{17},D_{19},D_{21},D_{22},D_{24}\}$
	C_3	$\{D_8,D_{25}\}$
MBWT	C_1	$\{D_1,D_2,D_5,D_6,D_7,D_8,D_{13},D_{18},D_{20},D_{21},D_{23},D_{25}\}$
	C_2	$\{D_3,D_4,D_9,D_{10},D_{11},D_{15},D_{16},D_{17},D_{19},D_{22},D_{24}\}$
	C_3	$\{D_{12}\}$

Table 9: Classification rates of MHEWT and MHDWT with respect to MBWT when lag value = 30

Class	MHEWT	MHDWT
1	85.71	71.43
2	100.00	100.00
3	25.00	50.00
4	0.00	100.00
5	20.00	80.00
6	0.00	75.00
Average	38.45	79.41

Table 10: Classification rates of MHEWT and MHDWT with respect to MBWT when lag value = 50

Class	MHEWT	MHDWT
1	100.00	75.00
2	100.00	100.00
3	0.00	0.00
Average	66.67	58.33

CONCLUSION

The present study investigated extraction of skeleton primitives by using skeleton subset. Though it is a tedious process of applying 2^8 possibilities, it is applied and it has given a clear and accurate dominant skeleton subset. If one wants classification mainly based on skeleton subset combination, then first method of classification is more appropriate. Since all textures considered have the same dimension second method of classification is also adopted based on least skeleton points. This classification may be appropriate for shape primitive classification. By increasing lag value, number of classes will be decreased and more number of textures will be added into each class. The present method of comparison of classification rates between different nonlinear wavelets by assuming one of them as standard one is also suited for other type of texture comparison based on different classification methods. The same method can be extended to 5×5, 7×7…N×N masks also.

REFERENCES

1. Mallat, S., 1989. Multifrequency channel decomposition of images and wavelet models. IEEE Trans. Acoustic, Speech and Signal Process., 37:2091-2110.doi=10.1109/ICSMC.1994.400223.
2. Laine A. and J. Fan, 1993. Texture classification by wavelet packet signatures. IEEE Trans. Pattern Anal. Machine Intel., 15: 1186-1191.doi= 10.1109/IAI.2000.839620.
3. Lu, C., P. Chung and C. Chen, 1997. Unsupervised texture segmentation via wavelet transform. Pattern Recognition, 30: 729-742. http://cat.inist.fr/ ?aModele= afficheN&cpsidt=2660327.
4. Arivazhagan, S. and L. Ganesan, 2003. Texture classification using wavelet transforms. Pattern Recognition Letters, 24: 1513-1521. http://www.ingentaconnect.com/content/els/01678 655/2003/00000024/00000009/art00390.
5. Shutao Li and James T. Kwok, 2003. Texture classification using support vector machines. Pattern Recognition Lett., 36: 2883-2893. DOI: doi:10.1016/S0031-3203(03)00219-X .
6. Li Wang and Dong-chen He, 1990. Texture classification using texture spectrum. Pattern Recognition, 23: 905-910. doi=10.1016/0031-3203(90)90135-8.

7. Eswara Reddy, B., A. Nagaraja Rao, A. Suresh and V. Vijaya Kumar, 2007. Texture classification by simple patterns on edge direction movements. IJCSNS, 7: 220-225. http://paper.ijcsns.org/07_book/200711/20071134.pdf.

8. Krishna, V.V., V. Vijaya Kumar, U.S.N. Raju, B. Saritha, 28th-30th October 2005. Classification of textures based on distance function of linear patterns using mathematical morphology. Proceedings of ICEM, Vol.4. Conducted by JNT University, India. http://www.vedamsbooks.com/no44382.htm.

9. Raju, U.S.N., V. Vijaya Kumar, A. Suresh and M. Radhika Mani, 27th – 30th May 2008. Texture description using different wavelet transforms based on statistical parameters. In: Proceedings of the 2nd WSEAS International Symposium on Wavelets Theory and Applications in Applied Mathematics, Signal Processing and Modern Science (WAV '08), Istanbul, Turkey, pp: 174-178. http://portal.acm.org/citation.cfm?id=1404119.

10. Suresh, A., U.S.N. Raju, A. Nagaraja Rao and V. Vijaya Kumar, 2008. An innovative technique of marble texture description based on grain components. Int. J. Comput. Sci. Network Sec., 8: 122-126. http://paper.ijcsns.org/07_book/200802/20080216.pdf.

11. Vijaya Kumar, V., B. Eswara Reddy, U.S.N. Raju and A. Suresh, 2008. Classification of textures by avoiding complex patterns. J. Comput. Sci., 4(2): 133-138. http://www.scipub.org/fulltext/jcs/jcs42133-138.pdf.

12. Vijaya Kumar, V., B. Eswara Reddy, U.S.N. Raju and K. Chandra Sekharan, 2007. An innovative technique of texture classification and comparison based on long linear patterns. J. Comput. Sci., 3: 633-638. http://www.scipub.org/fulltext/jcs/jcs38633-638.pdf.

13. Vijaya Kumar, V., B. Eswara Reddy and U.S.N. Raju, 2007. A measure of patterns trends on various types of preprocessed images. IJCSNS, 7: 253-257. http://paper.ijcsns.org/07_book/200708/20070836.pdf.

14. Vijaya Kumar, V., U.S.N. Raju, K. Chandra Sekaran and V.V. Krishna, 2008. A new method of texture classification using various wavelet transforms based on primitive patterns. ICGST Int. J. Graphics, Vision Image Process., GVIP, 8: 21-27. http://www.icgst.com/gvip/Volume8/Issue2/ P1150819004.pdf.

15. Loncaric, S., 1998. A survey of shape analysis techniques. Pattern Recognition, 31: 983-1001. doi=10.1.1.24.138.

16. Lam, L., S.W. Lee and C.Y. Suen, 1992. Thinning methodologies a comprehensive survey. IEEE Trans. Pattern Anal. Mach. Intell., 14: 869-885. doi= 10.1109/34.161346.

17. Smith, R.W., 1987. Computer processing of line images: A survey. Pattern Recognition, 20: 7-15. http://portal.acm.org/citation.cfm?id=31216.

18. Chang, H.-S., Yan, H., 1999. Analysis of Stroke Structures of Handwritten Chinese characters. IEEE Trans. Sys., Volume 29, Issue 1, pp.47 – 61. doi= 10.1109/3477.740165.

19. Saint-Marc, P., Rom, H., Medioni, G.,1993. B-spline contour representation and symmetry detection. IEEE Trans. Pattern Anal. Mach. Intell. 15, pp.1191–1197. http://doi.ieeecomputer society.org/10.1109/34.244680.

20. Zou, J.J. and H. Yan, 2001. Skeletonization of ribbon-like shapes based on regularity and singularity analyses. IEEE Trans. Syst. Man Cybern. 31: 391-395. doi= 10.1109/3477.931528.

21. Lam, L., S.W. Lee and C.Y. Suen. 1992. Thinning methodologies-a comprehensive survey. IEEE Trans. Pattern Anal. Machine Intel., 14: 869-885. doi= 10.1109/34.161346.

22. Bruekers, F.A.M.L. and A.W.M. van den Enden, 1992. New networks for perfect inversion and perfect reconstruction. IEEE J. Selected Areas Commun., 130-137. doi= 10.1109/49.124464.

23. Heijmans, H.J.A.M. and J. Goutsias, 2000. Nonlinear multiresolution signal decomposition schemes-Part II: morphological wavelets. IEEE Trans. Image Process., 9: 1897-1913. http://cat.inist.fr/?aModele=afficheN&cpsidt=1522549.

24. Claypoole, R.L., Jr. Baraniuk, R.G. Nowak, R.D. , May 12th -15th 1999. Adaptive wavelet transforms via lifting. Proceedings of IEEE International conference at Department of Electrical and Computer Engineering, Rice University, Houston, Texas., Vol.3, pp.1513-1516. doi= 10.1109/ICASSP.1998.681737.

25. Heijmans, H.J.A.M. and J. Goutsias, 1998. Morphology based perfect reconstruction filter banks. In: Proceedings of the IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis, Pittsburgh, Pennsylvania, October 6-9, pp: 353-356. doi= 10.1109/TFSA.1998.721434.

26. Sweldens, W., A.F. Lain and M. Unser, 1995. The lifting scheme: A new philosophy in biorthogonal wavelet constructions. Wavelet Appl. Signal Image Process. III, Proceedings of SPIE, 2569: 68-79. http://achille.research.bell-labs.com/cm/ms/who/wi m/papers/spie95.ps.gz

27. Sweldens, W., 1996. The lifting scheme: A custom-design construction of biorthogonal wavelets. Applied Comput. Harmonic Anal.,Vol. 3, 186-200. http://www.ingenta connect.com/ap/ha/1996/00000003/00000002/art00 015

28. Brodatz, P., 1966. Textures A Photographic for Artists and Designers. I[st] Edition, Dover, NY.

29. Hiremath, P.S. and S. Shivashankar, 2006. Wavelet based features for texture classification. GVIP J., 6: 55-58. http://www.icgst.com/gvip/Volume6/ Issue3 /P1150636002.pdf.