The Finite-Sample Bias of the Autoregressive Parameter Under Rank-based Estimation

Steven Cook

Department of Economics, University of Wales Swansea
Singleton Park, Swansea SA2 8PP, Wales

Abstract: Monte Carlo simulation is employed to examine the finite-sample properties of the estimated autoregressive parameter associated with the Dickey-Fuller and rank-based Dickey-Fuller unit root tests. While the downward bias associated with estimator under the Dickey-Fuller test has long been noted in the literature, the corresponding properties for the rank-based test have not been considered previously. The results presented show that in comparison to the DF test, increased bias is present for the rank-based test.

Key words: Unit Roots, Rank-Based Testing, Bias

INTRODUCTION

Following the seminal study of [1], testing of the unit root hypothesis using DF tests has become a familiar feature of applied research in econometrics and time series analysis. In its simplest form, a series y_t is said to be a unit root process if the coefficient $? = 1$ in the autoregressive representation below:

$$y_t = ?y_{t-1} + u_t \qquad (1)$$

where u_t is an *i.i.d.* error term with a zero mean and constant variance. Since the initial study of DF, a range of alternative unit root tests have been proposed to examine the unit root hypothesis. Amongst these is the rank-based DF test of [2]. The use of rank-based estimation has obvious appeal as it is recognised to result in robustness to a range of problems. However, the results of GH show also show the rank-based unit root test including an intercept term to exhibit greater power than the corresponding DF test. In this study the properties of rank-based unit root test are further examined. It is known that the estimator of the autoregressive parameter $?$ is subject to finite-sample (downward) bias under ordinary least squares estimation (OLS) when applying DF tests [3, 4]. In the present analysis, the finite-sample properties of the rank-based estimator of $?$ are considered.

DF and Rank-based DF Tests: Considering 'with intercept' specifications of unit root tests, the DF test examines the unit root hypothesis using the null H_0: $\beta = 1$ in the following regression:

$$y_t = a + \beta\, y_{t-1} + e_t$$
$$t = 1, \ldots, T \qquad (2)$$

where T denotes the sample size employed. The rank-based DF test proposed by GH simply replaces y_t with r_t, where r_t denotes the rank of y_t in $y_0, \ldots, y_T$. The unit hypothesis is therefore tested via the null H_0: $d = 1$ in the following regression:

$$r_t = ? + d\, r_{t-1} + v_t$$
$$t = 1, \ldots, T \qquad (3)$$

However, the OLS estimator $\hat{b}$ is known to be biased downwards in finite-samples. In the present analysis, the nature of any bias associated with $\hat{d}$ is examined.

Monte Carlo Experimentation: To examine the properties of alternative estimators of the autoregressive parameter, the unit root process of (1) is numerically simulated for a range of values of $?$ and T. The specific values employed are $? = \{0.80, 0.85, 0.90, 0.95, 0.99\}$ and $T = \{50, 100, 250, 500, 1000\}$. All experiments are performed over 25,000 replications using the GAUSS programming language. The error term u_t is generated as a standard normal variate using the RNDNS procedure. For each experimental design, the DF and rank-based DF tests of (2) and (3) are performed. In addition to noting the expected value of the estimators ß and d across replications, the expected values of the estimated standard errors, SE(.), of these estimators are also recorded. The results obtained from this analysis are presented in Table 1. It can be seen that as expected, downward bias is associated with $\hat{b}$. However, while the bias increases as $?$ approaches 1, the bias is smaller for larger sample sizes. For example, when $? = 0.95$, the bias is 0.097 when $T = 50$, but is only 0.04 when $T = 1000$.

Table 1: DF and Rank-Based DF Test Results

T	?	$E\left[\hat{b}\right]$	$E\left[\hat{d}\right]$	$E\left[SE\left(\hat{b}\right)\right]$	$E\left[SE\left(\hat{d}\right)\right]$
50	0.80	0.725	0.699	0.099	0.103
50	0.85	0.769	0.742	0.092	0.096
50	0.90	0.812	0.784	0.084	0.088
50	0.95	0.853	0.823	0.074	0.080
50	0.99	0.887	0.856	0.063	0.071
100	0.80	0.764	0.743	0.065	0.067
100	0.85	0.812	0.791	0.059	0.062
100	0.90	0.859	0.838	0.052	0.055
100	0.95	0.904	0.884	0.043	0.047
100	0.99	0.937	0.917	0.034	0.039
250	0.80	0.786	0.769	0.039	0.041
250	0.85	0.835	0.819	0.035	0.036
250	0.90	0.884	0.870	0.030	0.031
250	0.95	0.933	0.921	0.023	0.025
250	0.99	0.969	0.959	0.015	0.018
500	0.80	0.793	0.777	0.027	0.028
500	0.85	0.842	0.829	0.024	0.025
500	0.90	0.892	0.881	0.020	0.021
500	0.95	0.942	0.933	0.015	0.016
500	0.99	0.980	0.974	0.009	0.010
1000	0.80	0.796	0.781	0.019	0.020
1000	0.85	0.846	0.833	0.017	0.017
1000	0.90	0.896	0.886	0.014	0.015
1000	0.95	0.946	0.939	0.010	0.011
1000	0.99	0.985	0.982	0.005	0.006

Considering the results for the rank-based test, it is clear that for all experimental designs considered, $\hat{d}$ exhibits greater downward bias than $\hat{b}$. This is perhaps surprising given the higher power of the rank-based test, but is explained by the larger standard errors associated with the estimator.

CONCLUSION

In this study the finite-sample bias of the autoregressive parameter has been examined under the rank-based DF test of [2]. The results obtained showed the downward bias of the estimator associated with this test to exceed that noted previously in the literature for the DF test. The standard errors of the estimators were also noted, and it was found that the standard errors for the rank-based test exceeded those of the DF test. However,

despite underestimating the autoregressive parameter, the increased power of the rank-based test noted in the literature results can be explained by the large standard error, or variability, associated with the estimator.

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