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Hill Climbing Method Using Claus Model for Categorical Data

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Abstract: Problem statement: In this study, a locally unbiased test statistic based on the Claussian method was presented. **Approach:** In this respect, exploiting special structures of modified quadratic hill climbing method and the Claussian statistics were considered. **Results:** We derived a non-modified quadratic hill-climbing procedure. The performance of the model in testing the estimation of Latent data was shown. **Conclusion/Recommendations:** The result testing for serial Kalman suggested that, computational simplicity may outweigh any non-negative definite in terms of second derivative matrix.

Key words: Claussian method, modified quadratic, non-modified quadratic, latent data, non-negative

INTRODUCTION

In this study, we propose a new methodology for, structure and estimating the parameter of a nonlinear and non-Gaussian model using the quadratic hill climbing method under the influence of Claussian method. The objective of these research is to determine how well non-linear and non-Gaussian process can be analysis using non-quadratic hill climbing model and its characteristics in Claussian environment will predict the output of the process. Many authors have investigated the problem of non-linear and Gaussian model. These includes the studied of the effect of non-Gaussian on state space[1-5,7-14,16]. Effort were made to considered using modified quadratic hill climbing method to adjust for quadratic hill climbing method originally developed by Goldfield, Quandt and Trotter[6], in solving non-linear and non-Gaussian contaminated with non-negative definite second derivative. These authors did not, however incorporate the Claussian model in the adjustment of the quadratic hill climbing method originally developed by Goldfield, Quandt and Trotter[6], in solving non-linear and non-Gaussian contained with non-negative definite second derivative. This study, hereby propose a non-modified quadratic hill climbing method. Let y_t be the observed time where time is a measure of days and month. γ_t represent the corresponding latent variables, x_t are the pseudo observation and R_t is a matrix. Also Jorgensen[7] examined Poisson under the influence of exponential family but tail to incorporate it into the modified quadratic hill model.

The study is organized as follows; we present a review of the widely used modified quadratic hill climbing method and it non-modified vision as proposed. A class Claussian model was developed using Possion distribution and applied to non-modified quadratic hill climbing for estimation of latent data. Computation on data analysis were carried out for modified and non-modified quadratic hill climbing method is which the superior results of the non-modified quadratic hill climbing method results are shows below. While our model performs very well in the example, one should be aware that the statistical validity of our approach and of any approach that deals with an error problem, rest upon certain plausible but untestable assumption. Finally we present a logical conclusion.

MATERIALS AND METHODS

In the present research, a model was developed for the prediction of measure of dye, in the production of textile material in a stored mass of plastic container. The developed model was n-dimensional one. It allowed interpolation and intrapolation analysis and the rate of convergence. For the equation of state space model[1,3,12,15].

Quadratic hill climbing method from Claus approach: In order to evaluate the effect of modified quadratic hill climbing method as applied to non-linear and non-Gaussian model, its basic detail can be found in Goldfied, Quandt and Trotter[6]. Suppose we are interested in the non-linear and non-Gaussian model with density $\rho(y/\omega)$ with the efficient algorithms for a given guess of the solution of $\hat{\omega}$. The new guess in term of Newton Raphson method is given by:

$$\beta = \gamma - [\ddot{k}(\omega / y)\,|_{\omega=\gamma}]^{-1}\dot{k}(\omega / y)\,|_{\omega=\gamma} \tag{1}$$

Where:

$$\dot{k}(\omega / y) = \frac{\partial \log(P \cdot / \cdot)}{\partial \omega}$$

$$\ddot{k}(\gamma) = \frac{\partial^2 \log p(\cdot / \cdot)}{\partial \omega \ \partial \omega'}$$

$\dot{k}$ = Taken to be the conditional independent i.e., as probability of ω given y, then in line with its principal of:

$$\begin{aligned} \beta &= \gamma - [\ddot{k}(y / \omega)|_{\omega=\gamma} - \theta^{-1}]^{-1} \ (\dot{k}(y / \omega)|_{\omega=\gamma} - \theta^{-1}(\gamma - \lambda)) \\ \beta &= [\theta^{-1} - \ddot{k}(y / \omega)|_{\omega=\gamma}]^{-1} (\dot{k}(y / \omega)|_{\omega=\gamma} \ \ddot{k}(y/\omega)|_{\omega=\gamma} \ \gamma - \theta^{-1}\lambda) \end{aligned} \quad (2)$$

The expression in Eq. 2 follows the basic idea of Kalman fitter and hence (2) can be rewritten as:

$$\beta = [\theta^{-1} + p^{-1}]^{-1}[p^{-1}x + \theta^{-1}\lambda] \quad (3)$$

Where:

$$p = -[\ddot{k}(y / \omega)|_{\omega=\gamma}]^{-1}$$

and

$$x = \gamma + p\dot{k}(y / \omega)$$

The above methodology can only work well when the Hessian matrix is negative definite, otherwise, line search and non-modified quadratic hill climbing method can be introduce to neutralize the effect of non-negative definite.

Line search is a process whereby $\ddot{k}(y / \omega)^{-1}\dot{k}(y / \omega)$ is multiply by an appropriate scalar $0 < \theta \le 1$ in (1) and defining:

$$\beta' = \gamma + \theta(\beta - \gamma) \quad (4)$$

Which enable the new guess to be computed in a straightforward manner.

Goldfield, Quandt and Trotter[6] shows that the second expansion of k(γ) at $\dot{\gamma}$ is given by:

$$k(\dot{\alpha}) - (\alpha - \dot{\alpha})' k'(\dot{\alpha}) + \frac{1}{2}(\alpha - \dot{\alpha})\ddot{k}(\dot{\alpha})(\alpha - \dot{\alpha}) \quad (5)$$

Will attain the maximum in the region consisting of γ. The logical expression of the modified quadratic iterative scheme is given as follows :

- Let z be a positive parameter with a suitable value:

Compute $\beta = \lambda + z\left\|\dot{k}(\dot{\gamma})\right\|$

and

$$\beta = \begin{cases} \gamma - [\ddot{k}(\omega / y)|_{\omega=\gamma} - \delta I]^{-1} \ \dot{k}(\omega / y & ; \delta \ge 0 \\ \gamma\text{-}\ddot{R}(\omega/y)|_{\omega=\gamma} \ \ \dot{k}(\omega/y) & ; \delta < 0 \end{cases} \quad (6)$$

- The fast algorithm for applying quadratic Hill Climbing method is given as:

$$\begin{aligned} \tilde{y}_t &= \gamma_t + \varepsilon_t, \ \varepsilon_t \sim N(0, N), \\ \gamma_{t+1} &= g_t + L_t\gamma_t + S_t, \ S_t \sim N(0, Z_t) \end{aligned} \quad (7)$$

For a proof of the equivalence of β and the smoothed value details contact the author . Hence β can be obtained as the Kalman filter smoothed value of γ as given in (3). The standard approach for the model in (7) to estimate the state γ variable in nonlinear and non-Gaussian state space model is given by:

$$\gamma_{t+1} = f(\gamma_t) + S_t \quad (8)$$

using Taylor expansion about a point $\hat{\gamma}_t$:

$$f_t(\gamma_t) = f_t(\ddot{\gamma}_t) + \frac{\partial f_t(\gamma_t)}{\partial \gamma_t}|_{\gamma=\hat{\gamma}} (\gamma_t - \hat{\gamma}_t) \quad (9)$$

RESULTS AND DISCUSSION

Extension of quadratic hill climbing method to Claussian model: We establishes a relationship between the state space non-Gaussian model that are conditionally independent.

The state space model used in this study, evolving following time instead of row:

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} | \gamma_t \sim N\left\{\begin{pmatrix} Q_t^T \ \gamma_t \\ H_t \ \gamma_t \end{pmatrix}, \begin{bmatrix} \Psi_t & 0 \\ 0' & \tau_t \,|\, I_{T-1} \end{bmatrix}\right\} \quad (10)$$

$$P(\gamma_t) \ \alpha \ 1 \quad (12)$$

Let Z_t^T denote the matrix with time Z_t^T. The covariance becomes:

$$\begin{pmatrix} \gamma_t \\ x_t \end{pmatrix} | \begin{pmatrix} \gamma_t \\ \alpha \end{pmatrix} \sim N\begin{bmatrix} Q_t^T & Z_t \\ H & 0 \end{bmatrix} \begin{bmatrix} \gamma_t \\ \alpha \end{bmatrix}, \begin{bmatrix} \Psi_t & 0 \\ 0 & \tau_t \ I_{T-1} \end{bmatrix} \quad (13)$$

$$\begin{pmatrix} y_t \\ \alpha_t \end{pmatrix} | \begin{pmatrix} \gamma_{t-1} \\ \alpha \end{pmatrix} \sim N\left[\begin{pmatrix} \gamma_{t-1} \\ \gamma \end{pmatrix}, \begin{bmatrix} \tau_2^2 I_T & 0 \\ 0 & 0 \end{bmatrix}\right] \quad (14)$$

$$P(\gamma_t)\ \alpha\ 1 \quad (15)$$

$$P(\gamma_t) \sim N(M_\alpha,\ C_\alpha) \quad (16)$$

The expression above is similar to Poisson distribution. Let t time be denoted by ij where i represent days and j represent month, Y_{ij} denote univariate Poisson distribution. Conditional independent given the variable γ_{ij} and the vector.

Let Z_{ij}^T be the i days and j month in the matrix Z^T. Therefore, the model can be written as:

$$y_{ij} \mid (\gamma,\alpha) \sim P_0(\exp(Z_{ij}^T\alpha\ \ q_{ij}^T\ \ \gamma_t)) \quad (17)$$

Where:

$$\lambda_{ij} = \ xp(Z_{ij}^T\ \alpha + q_{ij}^T\gamma_i)$$

$$\mu_{ij} = Z_{ij}^T\ \alpha + q_{ij}\,\gamma_i$$

Have a Poisson distribution, while:

$$x_{ij} \mid (\gamma_{ij}\ ,\ \gamma_{i,j+1}) \sim N(\gamma_{ij} - \gamma_i, \gamma_{i,j+1}, \gamma_2^2) \quad (18)$$

is the Markov random field and:

$$\gamma_{ij} \mid \gamma_{i-1,j} \sim N(\gamma_{i-1}, \tau_2^2) \quad (19)$$

The Poisson distribution with $\lambda = \exp(\mu)$ has density:

$$P(y/\lambda) = \frac{\lambda^y \exp(-\lambda)}{y!} = \exp(y\log\lambda - \lambda - \log(y!))$$

hence

$$\log(p/\mu) = y\mu - \exp(\mu) - \log(y!) \quad (20)$$

Since an exponential family with a canonical link function is given by:

$$p(y_t \mid \mu_t) = \exp\{-a_t(\mu_t) + b_t(y_t)\}y_t^T\mu_t \quad (21)$$

Comparing (20) with (21) $b(\mu) = \exp(\mu)$, which is invariant under differentiation with respect to μ. Letting $\dot{\mu}$ be the point of Taylor expansion:

$$\frac{\partial \log p(y_t \mid \mu_t)}{\partial\ \mu_t} = y_t - \dot{a}_t - \ddot{a}_t(\mu_t - \hat{\mu}_t)$$

Let $\tilde{y} = \tilde{\mu}_t - \ddot{a}^{-1}(\dot{a}_t - y_t)$ and $\tilde{\Delta}_t = \ddot{a}_t^{-1}$, hence we have:

$$\tilde{\Delta} = \exp(-\mu) \quad (22)$$

$$\begin{aligned}\tilde{y} &= \tilde{\mu} - \exp(-\tilde{\mu})[\exp(\tilde{\mu}) - y] \\ \tilde{y} &= \tilde{\mu} + \tilde{\Delta}\ y - 1\end{aligned} \quad (23)$$

Let (23) be known as Claussian model. Thus $\tilde{y}/\mu \sim N(\mu, \tilde{\Delta})$ is a Gaussian approximation to the condition distribution $y/\mu \sim p_0(\exp(\mu))$.

Let $\tilde{\alpha}^{(0)}$ and $\tilde{\gamma}^{(0)} = \gamma_{ij}^{(0)}$ be initial values of the known variable and let:

$$\tilde{\mu}_{ij}^0 = Z_{ij}^T\ \tilde{\alpha}^{(0)} = Q_{ij}^T\ \tilde{\gamma}_i^{(0)}$$

Hence:

$$\Delta_{ij}^{(1)} = \exp(-\tilde{\mu}_{ij}^{(0)}) \quad (24)$$

$$\tilde{y}_{ij}^{(1)} = \tilde{\mu}_{ij}^{(0)} + \tilde{\Delta}_{ij}^{(1)}\ y_{ij} - 1 \quad (25)$$

The model (25) establish a Claussian state space model, which can be statistically represented as:

$$\tilde{y}_t = \tilde{\mu}_t + \tilde{\Delta}\ y_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, N_t) \quad (26)$$

Where:

$$\tilde{\mu}_t + \tilde{\Delta}\ y_t - 1 = \gamma_t$$

and

$$\gamma_{t+1} = g_t + L_t(\mu_t + \Delta\ y_t - 1) + S_t,\ S_t \sim N(0, R_t) \quad (27)$$

Which is the proposed non-modified quadratic hills climbing models.

Table 1: Hill climbing numerical application

Variable	Month	AIC	Kalman filter	Convergence
X_1	January	7.799040	119.81180	6
X_2	February	6.177289	94.74798	5
X_3	March	3.254279	49.44598	4
X_4	April	4.363627	66.63622	4
X_5	May	6.890154	105.79740	5
X_6	June	3.882320	59.17596	3
X_7	July	3.850472	50.93231	4
X_8	August	3.763260	57.33053	4
X_9	September	3.468264	52.75810	4
X_{10}	October	3.872189	59.01893	4
X_{11}	November	3.531503	53.73829	3
X_{12}	December	3.703706	56.40745	4

Since the model (26) and (27) are already in the linear form. We use second approximation on S_t, therefore:

$$l_{s_t}(x) = \log p_{s_t}(x)$$

where, p_{s_t} is the distribution of S_t. The numerical application are as shown in Table 1.

The data analysis for non-modified and modified quadratic hill climbing methods were carried out separately using data obtained from a textile manufacture industry in Nigeria and the following results were obtained using Claussian model. The result of non-modified quadratic hill climbing method had improved in convergence when compare to the modified quadratic hill climbing method; due to the fact that convergence was first obtained only after the 6th iterative process for modified quadratic hill climbing method and that of non-modified quadratic will climbing method is as shown in the column "convergence".

CONCLUSION

This piece of research had incorporated the Claussian model in the adjustment of the quadratic hill climbing method.

In conclusion, modified quadratic till climbing method and its application area have undergone majors developments in this article and we expect that they will continue to grow, as statisticians become fully aware of their important and the tool for performing statistical analysis and note, the challenges of developing new tools.

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