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Mathematical Model for Waste Reduction in Aluminum Fabrication Industry in Kuwait

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Abstract: Problem statement: Waste generation in the aluminum industry throughout the fabrication processes in Kuwait. **Approach:** A mathematical model has been developed to analyze the fabrication process and a special heuristic is designed for solving the model. The model uses actual data presented from an Aluminum Fabrication Industry (AFI). **Results:** Reduced the amount of waste generated substantially during the process. **Conclusion/Recommendations:** Considerable savings in waste generated can be realized by using scientific approaches through mathematical modeling.

Key words: Profile, fabrication, heuristic, mathematical model

INTRODUCTION

Aluminum is widely used worldwide in many forms. Houses, buildings and shops use aluminum made windows and doors produced by fabrication industries. These industries use aluminum profiles to manufacture different products. The profiles are produced through aluminum extrusion process and are made in various shapes, sizes and colors. In the Aluminum Fabrication Industry (AFI), profiles are cut into desired lengths to produce various products such as doors and windows. Waste is produced as a by-product; it constitutes 10% of the aluminum used in AFI.

Profile cutting for fabrication has been studied thoroughly with the objective of finding ways of reducing the amount of waste generated. A detailed mathematical model was built for this purpose. A heuristic has been developed for solving this model and it was tested on data for 350 windows and found to produce significantly less waste than the current conventional technique in use. The computational study involved in the process is presented. Results and recommendation is included. A step-by-step calculation of the amount of waste generated using the proposed heuristic for a specific profile is given.

The amount of waste generated is usually dependent on the profile cutting process used. Stock Cutting Problem (SCP) is discussed thoroughly in the literature. Gilmore and Gomory (1961) discussed the linear programming approach to the cutting stock problem. They suggested that its expression as an integer programming problem, involves a large number of variables, which generally makes computation infeasible. The difficulty presented by the enormous number of columns was overcome by solving a knapsack problem at every pivot step. This approach enabled to compute with a matrix which never has more columns than rows. Gilmore and Gomory (1964) discussed the cutting stock problems involving two or more dimensions and dealt with a wide range of industrial problems, especially those related to multistage cutting.

Haessler (1971) described a heuristic procedure for scheduling the production-rolls of study through a finishing operation to cut them down to finished roll sizes. The objective was to minimize the cost of trim-loss and reprocessing. The procedure generates cutting patterns and uses levels sequentially until the requirements are satisfied. At each step, the search depends upon the characteristics of the unsatisfied requirements. A maximum number of three solutions are generated for each problem. If none satisfy a predetermined aspiration level, the best of the three is chosen. Coverdale and Wharton (1976) presented a heuristic procedure for a nonlinear cutting stock problem; the article deals with scheduling cutting operations introduced the difficulties associated with selecting a few cutting patterns from a vast number of feasible options such that the total cost is minimized. The problem was solved using the pattern enumeration technique. However, the problem’s structure differs from the one presented in this study which solves nonlinearity of product form.

Adamowicz and Albano (1976) presented a method for solving aversion of the two-dimensional cutting stock problem. One is given a number of rectangular sheets and an order for a specified number of each rectangular shape. The goal is to cut shapes out of the sheets in such a way so as to minimize the waste, without using excessive amount of computational time. The solution method utilizes a constrained dynamic programming algorithm to lay out groups of rectangular structures, called strips.

Christofides and Whitcock (1977) presented a tree search algorithm for a two-dimensional cutting problem, in which there is a constraint on the number of each piece to be produced. His algorithm limits the size of the tree search by deriving and imposing necessary conditions for optimizing the cutting pattern. A dynamic programming approach was used to solve the unconstrained problem and a node-evaluation procedure was used to produce upper bounds during the search.

Tokuyama and Uneo (1986) discussed the cutting stock problem for large pieces in the iron and steel industries. The industrial challenge was characterized by the existence of a large variety of criteria, such as maximizing yield and increasing efficiency of production lines and the cutting stock problem is accompanied by an optimal selection dilemma. A two phase algorithm was developed using a heuristic; it gives a near-optimal solution in real time and is applied to both batch-solving and on-lone solving of one-dimensional cutting large pieces.

Sumichrast (1986) addressed this issue by interpreting a scheduling problem in the woven fiber glass industry as an example of the cutting stock problem, where wasted production capacity rather than wasted material is to be controlled. A heuristic was produced for scheduling the production.

Vanderbeck (2000) proposed an integer programming formulation for the problem that involves an exponential number of binary variables and associated columns, each of which corresponds to selecting a fixed number of copies of a specific cutting pattern. The integer program was solved using a column generation approach where the subprogram is a non-linear integer program that can be decomposed into a multiple bounded integer program. Ragsdale and Zobel (2004) identified and discussed a new type of one-dimensional cutting stock problem called the ordered CSP, which explicitly restricts the number of jobs in a production process that can be opened, or processed, at any given point in time. A mathematical formulation is provided for the new CSP model its applicability is discussed with respect to a production problem in the custom door and window manufacturing industry. A Genetic Algorithm (GA) is used for reducing waste levels. Several production scenarios using GA were tested and computational results are provided.

Cui and Lu (2009) developed an algorithm that uses both recursive and dynamic programming techniques to solve a rectangular two-dimensional cutting stock problem in a steel bridge construction. Poldi and Arenales (2009) examined the classical one-dimensional integer stock cutting problem, they developed a heuristic in order to obtain a integer solution. The objective was to minimize waste generated from cutting the available stocks. Dikili *et al*. (2007) proposed a novel approach for solving a one dimensional cutting stock problem in ship building. They used cutting patterns obtained by the analytical methods and mathematical modeling stage. By minimizing both the number of different cutting patterns and material waste, they proposed method was able to capture the ideal solution of the analytical methods. Feng *et al*. (2002) used artificial neural network in metal cutting processes, while Al-Wedyan *et al*. (2001) used fuzzy modeling techniques for down milling cutting problem. Singh *et al*. (2002) illustrated the effectives of Taguchi method in stock cutting problem.

MATERIALS AND METHODS

Aluminum Profile Extrusion (APE): Aluminum profile extrusion process involves several stages such as:

- Castings: Pure aluminum ingots, aluminum waste and other additives are mixed in a furnace at specified temperature to produce logs. Some plants forego this stage by importing ready-made billets or logs.
- LOG cutting: Each log is cut into standard billets according to demand. Extrusion: Billets are passed through an extrusion machine where profiles of different types and shapes are produced according to orders.
- Aging: The extruded aluminum profiles are placed into an aging furnace in order to increase its strength and durability.
- Polishing: At this stage profiles are thoroughly polished before being either anodized or painted
- Painting: Profiles are painted with the customers desired colors.
- Anodizing: Profiles are placed in anodizing tanks and colored according to customer's requests. The coloring must meet specifications, or the profile will be rejected and scrapped.

Table 1: Weight and percentage of total waste produced at AFI

Year	Aluminum profiles used per year (kg)	Waste weight per year (tons)	Percentage waste
1	1070	99	9.0
2	716	77	11.0
3	480	41	9.0
Mean	755	72	9.5
Standard deviation	297	29	

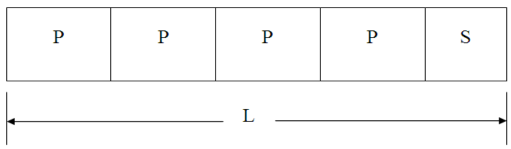

Fig. 1: Waste generated (s) by cutting a profile of length L into X pieces of length p

After the extrusion process, profiles are sent to different local or regional fabrication plants. The AFI consumes more than 100 tons/month of the aluminum produced by the local extrusion plant (AEC). At AFI, profiles are cut into different lengths to produce different products such as windows and doors. Large amounts of waste are generated at the fabrication process (Table 1).

In order to reduce the amount of waste generated, a mathematical model is developed a heuristic based on a stock cutting problem is produced and used (Fig. 1).

Mathematical model:

$$\text{Minimize } Z = \text{Minimze}\,[ML - pN]$$

Subject to:

$$ML \geq pN \quad (1)$$

$$L \geq pX \quad (2)$$

$$MX \geq N \quad (3)$$

$$L \leq pX \quad (4)$$

$$S \leq L \leq U \quad (5)$$

$$L, S, U \geq 0 \quad (6)$$

$$M, X \geq 0 \text{ and integers} \quad (7)$$

Where:

Decision variables:

L = Length of the profile

X = Number of pieces of the desired length in each profile

M = Number of profiles used

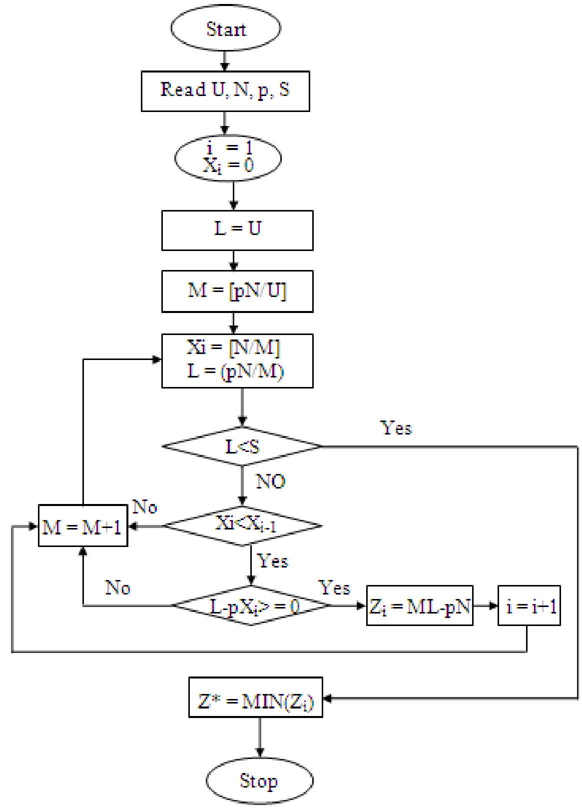

Fig. 2: Flowchart of the proposed heuristic

Input parameters:

p = Length of each piece

N = Total number of pieces of length p demanded (ordered)

U = Upper bound on the length of each profile

S = Lower bound on the length of each profile

Proposed heuristic: The heuristic initially takes the length of the profile to be as long as possible (U) and finds both the number of profiles to be ordered (M) for a specific demand the number of pieces produced by each profile (X) for a given piece-length (p). Keep reducing the profile length calculate X and M and the amount of waste produced (Z) for different profile lengths (L) above the lower bound S (cm) and below the upper bound U (cm). Find the minimum of all waste produce (Z^*) by different L, X, M. The L^*, X^*, M^* produced by the minimum Z value (Z^*) are the best values for the decision variables. A flow chart of the heuristic is given in Fig. 2; a step by step presentation of the heuristic is as follows:

Step 1: Let L=U

Step 2: M = [pN/U]

Table 2: Comparison of actual and predicted waste from cutting profiles for 80 windows at AFI

Sr. No.	Profile number	Length (p) of each piece (cm)	Number (N) of pieces needed	Profile Length (L) (cm) Conv.*	Heut+	Total scrap generated (cm) Z = (ML-pN) Conv.	Alg.
1	1440	61.7	160	600	620	805	48
2	1440	71.7	160	600	580	528	128
3	1440	59.7	160	600	600	54	48
4	2117	63.3	160	600	510	546	72
5	2117	23.7	320	600	640	163	96
6	2117	63.0	160	600	635	594	80
7	2117	41.5	160	600	670	228	60
8	2117	51.5	160	600	520	503	80
Total scrap						3421	612

*: Conventional techniques used at AFI; +: Proposed heuristic

Table 3: Comparison of actual and predicted waste from cutting profiles for 250 windows at AFI

Sr. No.	Profile number	Length (p) of each piece (cm)	Number (N) of pieces needed	Profile Length (L) (cm) Conv.*	Heut+	Total scrap generated (cm) Z = (ML-pN) Conv.	Alg.
1	2181	109.0	1000	575	660	4000	1210
2	2182	99.0	500	620	600	1660	900
3	2183	99.0	500		600	1660	900
4	2056	98.0	1000	610	590	3320	530
5	2057	98.0	1000	610	590	3320	530
6	2058	42.0	2000	610	590	1540	370
7	2059	45.0	1000	600	630	1155	360
8	2059	98.0	1000	600	590	3320	530
9	2060	103.5	1000	575	520	11000	500
Total scrap						30975	5830

*: Conventional techniques used at fabrication industry; +: Proposed heuristic

Step 3: $X_i = [M/N]$ if $X_i \le X_{i-1}$ increase M and calculate X_i until the new X_i is greater than the previous X_{i-1} i.e., $X_i \le X_{i-1}$.

Step 4: L = (pN/M)
L< S if yes go to 7, otherwise continue.

Step 5a: $L\text{-}pX_i \ge 0$ Go to step 6.

Step 5b: $L\text{-}pX_i \le 0$ Let M = M+1 and go to step 3.

Step 6: $Z_i = ML\text{-}pN$ Let M = M+1 and i = i +1, go to step 3.

Step 7: Z^* = minimum (Z_i) list L^*, M^*, X^* and stop.

[a] largest integer greater or equal to a.

The results of applying the above heuristic on the data provided by the AFI are shown in Table 2 and 3 which present the total amount of waste generated by the conventional method currently used by the industry compares it to the waste produced if the special heuristic is used.

Computational study: The proposed algorithm for profile cutting was implemented on two batches of 80 and 250 windows, respectively. Several profiles are used in producing each window. The profile type, the desired piece-length the quantity needed are presented in Table 1, along with the waste generated from cutting each profile by fabrication industry's conventional method and by the proposed heuristic. The heuristic obviously generates far less waste than the conventional method. A step-by step calculations of the waste generated when cutting profile number 1440 to provide a piece of 71.7 cm piece cutting profile number 2117 to produce a 23.7 cm piece is given in two examples.

Examples:

Example 1:

Profile Number 2117, p = 23.7 cm, N= 320 pieces

Step 1: L = 700cm
Step 2: M = [(23.7)(320)/700] = 11 profiles
Step 3: X = [(320)/11] = 30
Step 4: L = [(23.7)(320)/11] = 698.45 cm

Let L = 690 cm > 500 cm

Step 5: L-pX = 690- 23.7 (30) = - 21, M = M+1 = 11 +1 =12
Step 3: X = [(320)/12] = 27

Step 4: L = [(23.7)(320)/2 = 632 cm
Step 5: L-pX = 640- 23.7 (27) = 0.10
Step 6: Z = LM-pN = 7680-7584 = 96cm
Step 4: L = [(23.7)(320)/12] = 632 cm

Let L = 640 > 500

Step 5: L-pX = 640- 23.7 (27) = 0.10
Step 6: Z = LM-pN = 7680-7584 = 96cm, M = 13
Step 4: L = [(23.7)(320)/13] = 583.4 cm

Let L = 590 cm > 500 cm

Step 3: X = [(320)/13] = 25
Step 5: L-pX = 590- 23.7 (25) = -2.5, M= 14
Step 3: X = [(320)/14] = 23
Step 4: L = [(23.7)(320)/14] = 541.7 cm

Let L = 550 cm > 500 cm

Step 5: L-pX = 590- 23.7 (23) = 4.9
Step 6: Z = LM-pN = 116 cm, M = 15
Step 4: L = [(23.7)(320)/15] = 505cm

Let L = 510 cm > 500 cm

Step 3: X = [(320)/15] = 22
Step 5: L-pX = 51- 23.7 (22) = -11.4, M = M+1=15+1=16
Step 3: X = [(320)/16] = 20
Step 4: L = [(23.7)(320)/16] = 474.0

L= 470.0<500

Step 7: Z = Minimum (96,116) = 96 cm

L = 640
M = 12
X = 27

Example 2:

Profile Number 1440, p = 713.7 cm, N= 160 pieces

Step 1: L = 700cm
Step 2: M = [(71.7)(160)/700] = 171 profiles
Step 4: L = [(71.7)(160)/17] = 674.82 cm

Let L = 680 cm > 500 cm

Step 3: X = [(160)/17] = 10 pieces
Step 5: L-pX = 680- 17.7 (10) = - 37, M = M+1 = 17 +1 =19
Step 3: X = [(160)/18] = 9 pieces
Step 5: L-pX = 680- 71.7 (9) = 34.7
Step 6: Z = LM-pN = 768cm, M = M+1 = 18 +1 =18
Step 4: L = [(71.7)(160)/19] = 603.8 cm

Let L = 610 > 500

Step 3: X = [(160)/19] = 9 pieces, M = M+1 = 19+1 =20
Step 3: X = [(160)/20] = 9 pieces, M = M+1 = 19+1 =20
Step 4: L = [(71.7)(160)/20] = 573.86 cm

Let L = 580 cm > 500 cm

Step 5: L-pX = 580- 23.7 (20) = 6.4 cm
Step 6: Z = LM-pN = 11600-11472 = 128cm, M M+1= 20 + 1 =21
Step 4: L = [(71.7)(160)/21] = 546.3 cm

Let L = 550 cm > 500 cm

Step 3: X = [(160)/21] = 8, M = M+1 = 21+1 = 22
Step 3: X = [(160)/22] = 8, M = M+1 = 221+1 = 23
Step 3: X = [(160)/23] = 7, M = M+1 = 221+1 = 23
Step 4: L = [(71.7)(160)/23] = 498.8 cm
Step 5: L-pX = 590- 23.7 (25) = -2.5, M= 14
Step 3: X = [(320)/14] = 23
Step 7: Z = Minimum (768,128) = 128 cm

L = 580 cm
M = 20
X = 128 cm

RESULTS AND DISCUSSION

Table 2 and 3 present detailed information on cutting profiles for window fabrication at the AFI. Table 2 shows the amount of waste generated by the profiles used in producing the first batch of 80 windows. The conventional procedure produced 34.21 m whereas the heuristic produced 6.12 m, a difference of around 28 m. Table 3 shows the amount of waste generated by profiles used in fabricating 250 windows. The fabrication industry's conventional procedures produced 309.75 m, whereas the proposed heuristic algorithm produced 580.30 m, a difference of around 251.45 m of scrap.

CONCLUSION

The aluminum fabrication industries generate large amounts of scrap, mainly due to techniques used in cutting. An efficient optimal cutting method would not only minimize the amount of waste produced, but will also result in more efficient usage of time and manpower. In this study, a heuristic was developed that generates less waste than the current procedures. Table 1 demonstrates that the existing conventional procedures should be re-evaluated and replaced by scrap-minimization approaches. In profile number 2060 for example, the conventional method produced 11,000 cm of scrap, whereas the heuristic produced 500 cm. The average waste generated per window is around 1.4m, with fabrication industry conventional techniques, but only about 0.196 m using the heuristic. Since the fabrication industry produces an average of around 12,000 windows annually, the amount of waste generated is around 12,480 meters using the existing method about 2,352 meters using the heuristic, which would thus save around 10,000 m annually. Clearly, the fabrication industry's waste level is unnecessarily high the fabrication procedure should be improved. The proposed heuristic could replace the currently used techniques. Given an order for windows in terms of type, size quantity, the proposed procedure can be used to determine:

- Profile length from each type required
- Number of profiles of each type required
- Number of billets of each length required
- Number of logs of each length to be cut

On the basis of the study presented in this study, considerable savings in waste can be realized by applying mathematical models and computer-based optimization procedures.

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