

Indirect Kalman Filter in Mobile Robot Application

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Abstract: Problem statement: The most successful applications of Kalman filtering are to linearize about some nominal trajectory in state space that does not depend on the measurement data. The resulting filter is usually referred to as simply a linearized Kalman filter. **Approach:** This study introduced mainly indirect Kalman filter to estimate robot's position. A developed differential encoder system integrated accelerometer is experimental tested in square shape. **Results:** Experimental results confirmed that indirect Kalman filter improves the accuracy and confidence of position estimation. **Conclusion:** In summary, we concluded that indirect Kalman filter has good potential to reduce error of measurement data.

Key words: Indirect Kalman filter, differential encoder system, sensor data fusion

INTRODUCTION

Kalman's study of the early 1960s was recognized almost immediate as new and important contributions to least-squares filtering. As a result, there was a renewal of research interest in this area. Research study in this area still continues and new applications and extensions continue to appear regularly in the technical literature (Aggarwal *et al.*, 2005). Some of the most successful applications of Kalman filtering have been in sit nations with nonlinear dynamics or nonlinear measurement relationships. One is to linearize about some nominal trajectory in state space that does not depend on the measurement data. The resulting filter is usually referred to as simply a linearized Kalman filter. The other method is to linearize about a trajectory that is continually updated with the state estimates resulting from the measurements. This filter is called an extended Kalman filter. Its use in the analysis of visual motion has been documented frequently. The standard Kalman filter derivation is given here as a tutorial exercise in the practical use of some of the statistical techniques (Elfes, 1987; Borenstein and Feng, 1996). Documenting this derivation furnishes the reader with further insight into the statistical constructs within the filter. The filter is constructed as a mean squared error minimization, but an alternative derivation of the filter is also provided showing how the filter relates to maximum likelihood statistics. The purpose of filtering is to extract the required information from a signal, ignoring everything else. How well a filter performs this task can be measured using a cost or loss function. Indeed we may define the goal of the filter to be the minimization of this loss function (Norsuzila *et al.*, 2008; Gao *et al.*, 2007).

MATERIALS AND METHODS

The Kalman filter has long been regarded as the optimal solution to many tracking and data prediction tasks (Figueroa and Mahajan, 1994; Gelb, 1974; Haykin, 1996).

Indirect Kalman filter concept: The process to be estimated and the measurement relationship is written in the form:

$$\dot{x} = f [x, u_d, t] + w_{(t)} \quad (1)$$

$$z = h [x, t] + v_{(t)} \quad (2)$$

Where:

f and h = Known function

u_d = A deterministic forcing function

w and v = White noise with zero cross-correlation

From the truth trajectory $x_{(t)}^{Ref}$ is referred to as the nominal or reference trajectory and the actual trajectory $x_{(t)}$ can write as:

$$x_{(t)} = x_{(t)}^{Ref} + \delta x_{(t)} \quad (3)$$

From Eq. 1-3, then become:

$$\dot{x}_{(t)}^{Ref} + \delta \dot{x}_{(t)} = f(x^{Ref} + \delta x, u_d, t) + w_{(t)} \quad (4)$$

$$z = h(x^{Ref} + \delta x, t) + v_{(t)} \quad (5)$$

and with Taylor's series expansion, then the result, retaining only the first-order terms is:

$$\dot{x}^{Ref} + \delta\dot{x} \approx f(x, u_d, t) + \left[\frac{\partial f}{\partial x} \right]_{x=x^{Ref}} \bullet \Delta x + w_{(t)} \quad (6)$$

$$z \approx h(x^{Ref}, t) + \left[\frac{\partial h}{\partial x} \right]_{x=x^{Ref}} \bullet \Delta x + v_{(t)} \quad (7)$$

Where:

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_m} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \dots & \dots & \frac{\partial f_n}{\partial x_m} \end{bmatrix}$$

$$\frac{\partial h}{\partial x} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \dots & \frac{\partial h_1}{\partial x_m} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \dots & \frac{\partial h_2}{\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_n}{\partial x_1} & \dots & \dots & \frac{\partial h_n}{\partial x_m} \end{bmatrix}$$

From the truth trajectory $x_{(0)}^{Ref}$ to satisfy the deterministic differential equation:

$$\dot{x}^{Ref} = f(x^{Ref}, u_d, t) \quad (8)$$

Substituting this into Eq. 6 then leads to the linearized model:

$$\delta\dot{x} = \left[\frac{\partial f}{\partial x} \right]_{x=x^{Ref}} \bullet \Delta x + w_{(t)} \quad (9)$$

$$(z - h [x^{Ref}, t]) + \left[\frac{\partial h}{\partial x} \right]_{x=x^{Ref}} \bullet \Delta x + v_{(t)} \quad (10)$$

From Eq. 9 and 10 may be written in the discrete-time form. For process state equation is written as:

$$\delta x_{k+1} = F_k \delta x_k + \Gamma w_k \quad (11)$$

and measurement equation as:

$$\delta z_k = H_k \delta x_k + \Lambda v_k \quad (12)$$

From the basic linearized measurement equation, Eq. 10 the measurement presented to the Kalman filter

is $(z - h [x^{Ref}, t])$. Then the incremental estimate update equation at time t_k is considered as:

$$\delta\hat{x}_k^+ = \delta\hat{x}_k^- + K_k \bullet (z_k - h [x_k^{Ref}, t] - H_k \delta\hat{x}_k^-) \quad (13)$$

From above equation can consider term $z_k - h [x_k^{Ref}, t]$ as δz_k then the equation become:

$$\delta\hat{x}_k^+ = \delta\hat{x}_k^- + K_k \bullet (\delta z_k - H_k \delta\hat{x}_k^-) \quad (14)$$

The term $(\delta z_k - H_k \delta\hat{x}_k^-)$ is named as Residual and the matrix K_k is Kalman gain. It can minimize the posteriori covariance of the error estimate. The matrix K_k is calculated by:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + \Lambda_k R_k \Lambda_k^T)^{-1} \quad (15)$$

R_k is the measurement noise covariance at step k . The covariance of the priori estimate of δx is calculated as:

$$P_k^- = E \left[(\delta x_k - \delta\hat{x}_k^-) (\delta x_k - \delta\hat{x}_k^-)^T \right] \quad (16)$$

The posteriori estimate is:

$$P_k^+ = E \left[(\delta x_k - \delta\hat{x}_k^+) (\delta x_k - \delta\hat{x}_k^+)^T \right] \quad (17)$$

The equation of extend Kalman Filter for "Measurement update" are:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + \Lambda_k R_k \Lambda_k^T)^{-1} \quad (18)$$

$$\delta\hat{x}_k^+ = \delta\hat{x}_k^- + K_k \bullet (\delta z_k - H_k \delta\hat{x}_k^-) \quad (19)$$

$$P_k^+ = P_k^- - K_k H_k P_k^- \quad (20)$$

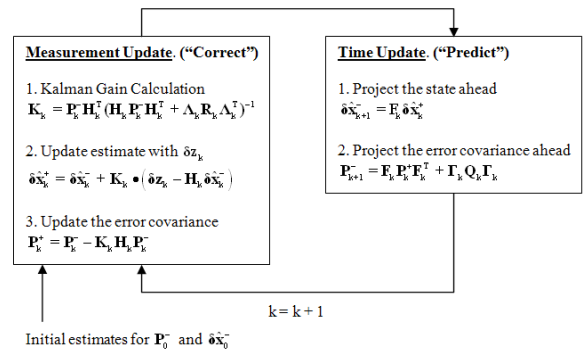


Fig. 1: Step of measurement update

and the equation of extend Kalman-Filter for “Time update” with the assumption the projection equation is not affected by the cross-correlation between system noise and measurement noise because of the whiteness property of each:

$$\delta \hat{x}_{k+1}^- = F_k \delta \hat{x}_k^+ \quad (21)$$

$$P_{k+1}^- = F_k P_k^+ F_k^T + \Gamma_k Q_k \Gamma_k^T \quad (22)$$

Generally for the measurement and time update step in the literature start at time t_k first the “Time update” and then the “Measurement update” shown in Fig. 1.

RESULTS

The experiment is carried out and presented in to demonstrate the feasibility, accuracy and performance using Kalman filter algorithm. The experiment focused on the observation of the position accuracy.

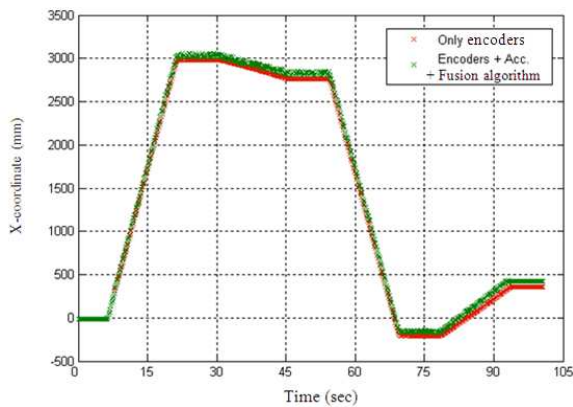


Fig. 2: Simulation of mobile robot’s position proposed system in X-coordinate with MATLAB software

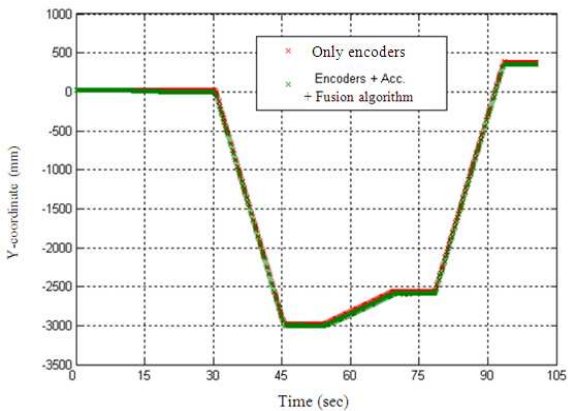


Fig. 3: Simulation of mobile robot’s position proposed system in Y-coordinate with MATLAB software

We made the experiment by using mobile robot driven by the differential encoders system and run mobile robot in square shape. In the first round the mobile robot only run by the differential encoders system. And in the second round the mobile robot run by the differential encoders system integrated with accelerometer and using indirect Kalman filter to reduce position error. The simulation of mobile robot’s position results is shown in Fig. 2 and 3 in X and Y coordinates.

DISCUSSION

The results will be compared and discussed. We start with calibration the accelerometer and controlled the vehicle along a square shape for $3 \times 3 \text{ m}^2$ with start point (0, 0). The trajectory was estimated based on shaft encoders from start point (0, 0) → point (0, 3) → point (3, -3) → point (0, -3) → end point (0, 0). The estimation error from fusion algorithm between encoders and accelerometer at end point in the X-coordinate is 23.5 cm and in Y-coordinate is 32 cm. We see that the estimation with sensor fusion can improve performance of mobile robot’s localization.

CONCLUSION

We have now presented the basics of Kalman filtering and looked at a few examples of how the technique can be applied in physical situations.

The Kalman filter is intended to be used for estimating random processes. The Kalman filter is a linear estimator. The filter is optimal in the minimum mean-square-error sense within a class of all estimators, linear and nonlinear. Under certain special circumstances, the Kalman filter yields the same result obtained from deterministic least squares. Kalman filtering is especially useful as an analysis tool in off-line error analysis studies. The optimal filter error covariance equation can be propagated recursively without actual measurement data.

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REFERENCES

Aggarwal, K.K., Y. Singh, P. Chandra and M. Puri, 2005. Bayesian regularization in a neural network model to estimate lines of code using function points. *J. Comput. Sci.*, 1: 505-509. <http://www.scipub.org/fulltext/jcs/jcs14505-509.pdf>

- Borenstein, J. and L. Feng, 1996. Measurement and correction of systematic odometry errors in mobile robot. *IEEE Trans. Robot. Automat.*, 12: 869-880. DOI: 10.1109/70.544770
- Elfes, A., 1987. Sonar based real-world mapping and navigation. *IEEE J. Robot. Automat.*, 3: 249-265. DOI: 10.1109/JRA.1987.1087096
- Flgueroa, F. and A. Mahajan, 1994. A robust navigation system for autonomous vehicles using ultrasonic. *Control Eng. Pract.*, 2: 49-59. DOI: 10.1016/0967-0661(94)90573-8
- Gao, X.W., S. Anishenko, D. Shaposhnikov, L. Podlachikova and S. Batty *et al.*, 2007. High-precision detection of facial landmarks to estimate head motions based on vision models. *J. Comput. Sci.*, 3: 528-532. <http://www.scipub.org/fulltext/jcs/jcs37528-532.pdf>
- Gelb, A., 1974. *Applied Optimal Estimation*. 1st Edn., MIT Press, Massachusetts, ISBN: 10: 0262570483, pp: 382.
- Haykin, S., 1996. *Adaptive Filter Theory*. 3rd Edn., Prentice Hall Inc., Englewood Cliffs NJ., ISBN: 10: 013322760X, pp: 989.
- Norsuzila, Y., M. Abdullah and M. Ismail, 2008. Leveling process of Total Electron Content (TEC) using Malaysian Global Positioning System (GPS) data. *Am. J. Eng. Applied Sci.*, 1: 223-229. <http://www.scipub.org/fulltext/ajeas/ajeas13223-229.pdf>