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Graph Representation for Secondary System of Pressurized Water Reactor with Autocatalytic Set Approach

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Abstract: The pace of nuclear reactor development in increasing and this is caused by the high authoritative ordinance of electricity from the end users. Furthermore, maintaining and controlling nuclear reactor operation are high priorities due to safety reasons. The aim of this paper is to describe the secondary system of Pressurized Water Reactor (PWR) system by using graph. Furthermore, fuzzy graph is used in performing a new graph based on its dynamicity. Finally, the results between crisp and fuzzy graph of the system are compared and verified against published data. The fuzzy graph of a secondary system of pressurized water reactor is better than a crisp graph of the system.

Keywords: Graph Theory, Nuclear Power Plant, Pressurized Water Reactor (PWR), Autocatalytic Set, Fuzzy Graph

Introduction

The Pressurized Water Reactor (PWR) contains two systems namely primary and secondary system as shown in Fig. 1. Nuclear fuel for PWR is Uranium dioxide (UO2) pellets located in the reactor core vessel of the primary system.

Most of the radioactive processes circulated within the primary system are caused by two separate systems of moderators circulating in steam generator. The aim of this paper is to model the process which occurs in the secondary system of PWR.

The phase change from water to steam occurs in a steam generator. The steam is transferred to turbine in order to generate electricity. Next, phase change from steam to water or condensation process occurs in main condenser, which will later be transferred back into the steam generator. This process is repeated until the PWR reactor is shut down (USNRC, 2011).

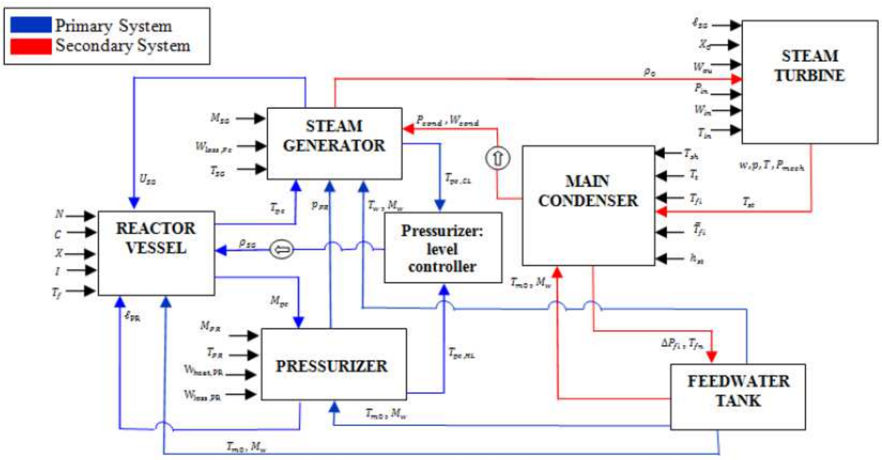

Fig. 1. Pressurizer Water Reactor (PWR) schemetic diagram system

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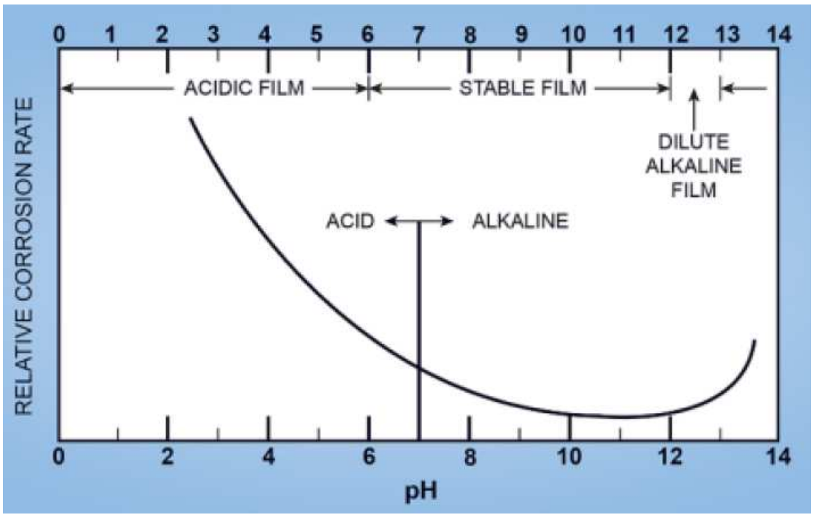

Fig. 2. Relationship between corrosion rate and pH for iron

Table 1. Incidents caused by corrosion

Plant	Year	Descriptions
Oconee 3	1976	Pinhole leak at extraction line
	1980	Thinned elbow caused by erosion/corrosion has been replaced
Browns ferry 1	1982	Failure of 8-inch discharge line at the MSR drain pump
Oconee 2	1983	Failure of a 3 to 10-inch expander downstream of a reheater drain tank
Calvert cliffs1	1984	Rupture of a 16-inch elbow in a branch line from a cold reheater steam line
Haddam neck	1985	Pipe rupture downstream of a feedwater heater
Kewaunee	1985	Rupture of a 2-inch excess steam vent line from a MSR
Hatch 2	1986	Rupture of a 20 to 16-inch reducer in an extraction steam line
Ginna	1986	Failures of a 6-inch elbow of a moisture separator reheater drain line

Castelli (2009) presented the corrosion rate over time of PWR. In other work, Wu (1989) stated that erosion/corrosion is due to oxygen concentration and pH rate. Gedeon (1993) presented that corrosion occurs when there is acid or low pH rate. The relationship between corrosion rate and pH for Iron is shown in Fig. 2. Table 1 present various accidents which have occurred as a result of corrosion as stated in (Wu, 1989).

Kokaji (1995) stated that most nuclear power plants in Japan utilize a great amount of sea water for cooling condenser. Ibrahim *et al*. (2014; Ibrahim and Badawy, 2014) presented the impact of seawater salinity on the performance of the condenser. The increasing of sea water salinity leads to the decreasing of the heat-transfer coefficients in the condenser. Frepoli (2008) presented a sample result of a typical large break for Loss Of Coolant Accident (LOCA) analysis of PWR.

Most cases which occur in the secondary system are caused by the chemical of the compound mixture in the system. Hence, this paper describes the process and the dynamic of the secondary system via graph representation. The vertices and edge of a graph are used to represent the compounds and chemical reaction in the secondary system of PWR. Furthermore, the concept of graph is explained and described in the next section. The result for implementations, of Autocatalytic Set (ACS) and Fuzzy Autocatalytic Set (FACS) for secondary system of PWR is presented in the final section.

Graph Theory

Graph theory has been widely used in many areas such as in physics, chemistry, psychology, sociology and computer sciences (Balakrishnan and Ranganathan, 2012). Graph is defined as the networks of points or nodes that are connected by links (Balakrishnan and Ranganathan, 2012). A directed graph $G = G(V, E)$ is defined by a set V of "nodes" and a set E of "links" where each link is an ordered pair of nodes. The set of nodes and links can be represented as $V = \{v_1, v_2, v_3, \ldots v_n\}$ and $E = \{e_1, e_2, v_3, \ldots\ v_n\}$with $C = (C_{ij})$ is the adjacency matrix of a graph.

On the other hand, the concepts of fuzzy is used and implemented into graph theory after the evolution of fuzzy set theory introduced by (Zadeh, 1969). A fuzzy graph defined by Rosenfeid (2014) is presented as follows:

Definition 1: Fuzzy graph G = (σ, μ) is a pair of functions $\sigma = S \rightarrow [0,1]$ and, $\sigma = S \times S \rightarrow [0,1]$, $\forall x, y \in S$ where $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

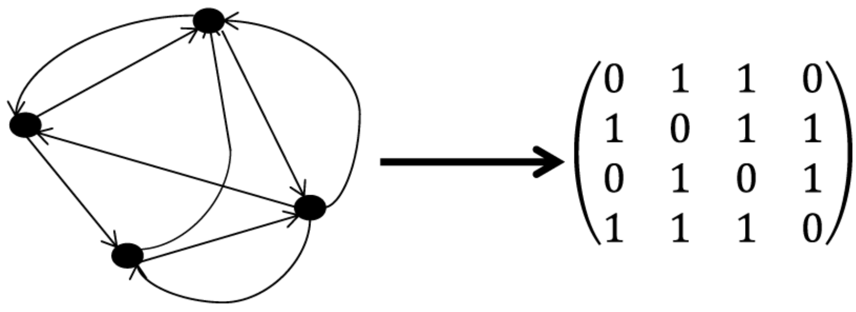

Fig. 3. A cycle, an Autocatalytic set (ACS) with 4 vertices

However, Tahir *et al.* (2009) introduced five types of fuzziness for graph based on the taxonomy of fuzzy graphs by Blue *et al.* (2002) as follows:

Definition 2: Fuzzy graph is a graph G_F which satisfies one of the fuzziness (G_F^i of the i^{th} type) or any of its combinations:

Type 1: $G_F^1 = \left\{G_{1_F}, G_{2_F}, G_{3_F}, ..., G_{n_F}\right\}$ = $\{V, E_F\}$ where fuzziness is on G_{i_F} for $i = 1,2,3,...n$.

Type 2: $G_F^2 = \{V, E_F\}$ where the edge set is fuzzy.

Type 3: G_F^3 $\{V, E\ (t_F, h_F)\}$ where for the vertex and edge set are crispy, but the edge has fuzzy head and tail.

Type 4: G_F^4 $\{V_F, E\}$ where the vertex set is fuzzy.

Type 5: G_F^5 $\{V, E\ (w_F)\}$ where for the vertex and edge set are crispy, but the edge has fuzzy weights.

The term autocatalytic refers to a product or compound used to speed up a chemical reaction known as catalyst. Ostwald (1894a; 1894b; 1894) stated that catalysis is used to accelerate the slow chemical reaction with an addition of a foreign substance, but it is not consumed by the reaction. In general, an autocatalytic set is defined as a set of entities or a collection of entities where the word entities can be anything such as people, molecule or object. Further, the concept of an autocatalytic set was introduced in the context of interaction between compounds presented by Jain and Krishna (1998) formally as follows:

Definition 3: An Autocatalytic Set (ACS) is a sub graph, each of whose nodes have at least one incoming link from a node belonging to the same sub graph (Fig. 3).

An autocatalytic set can be presented in the form of matrix $n \times n$. An autocatalytic set of an incineration process was introduced by Tahir *et al.* (2009) as follows:

Definition 4: Fuzzy Autocatalytic Set (FACS) is a sub graph each of whose nodes have at least one incoming link with membership value $\mu\ (e_i) \in (0,1]$, $\forall e_i \in E$.

These membership values are used in creating the adjacency matrix of a graph C.

An Autocatalytic Set of PWR System

A set of vertices $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ namely corrosion (v_1), moderator (v_2), sulphuric acid (v_3), boric acid (v_4), sodium hydroxide (v_5), chlorides (v_6) and nitrogen (v_7) play an important role in presenting a secondary system of PWR as shown in Fig. 4. Castelli (2009) stated that the compounds of Co, Zn, Ni, Fe and Zr are classified as corrosion products. However, in this study the term corrosion is used rather than corrosion products. The seven edges of E represent the links of vertices are shown in Table 2.

Fuzzy Autocatalytic Set of PWR System

The introduction of fuzziness into an autocatalytic set of fuzzy graph type 3 was given by Tahir *et al.* (2009):

Definition 5: Let $e_1 \in E$. The fuzzy head of e_i is denoted as $h(e_i)$ and the fuzzy tail $t(e_i)$ is a function of e_i such that h: $E \to [0,1]$ and t: $E \to [0,1]$ for $e_i \in E$. A fuzzy edge connectivity is a tuple $(t(e_i), h(e_i))$ and the set of all fuzzy edge connectivity is denoted as $C = \{(t(e_i), h(e_i)): e_i \in E\}$.

The value for $t(e_i)$ is equal to 1 since reaction with other variables is taken place. However, the value for $h(e_i)$ is taken in the interval of (0,1] due to the strength of connection or reaction with another variable. Therefore the membership value for fuzzy edge is defined as follows:

$$\mu(e_i) = min\ \{\ t(e_i), h(e_i)\} \quad (1)$$

The data for our models is taken from AP1000 nuclear power plant (Winters *et al.*, 2009a). The crisp graph G_s shows that each link has the same color and thickness due to the same value of connectivity between the vertices in the graph (Fig. 4(a)). However, the fuzzy graph G_{S_F} provided different membership value. The different colour of links signifies the distinct range of membership value where the greater value of connectivity is indicated by thickness of edges (Fig. 4(b)).

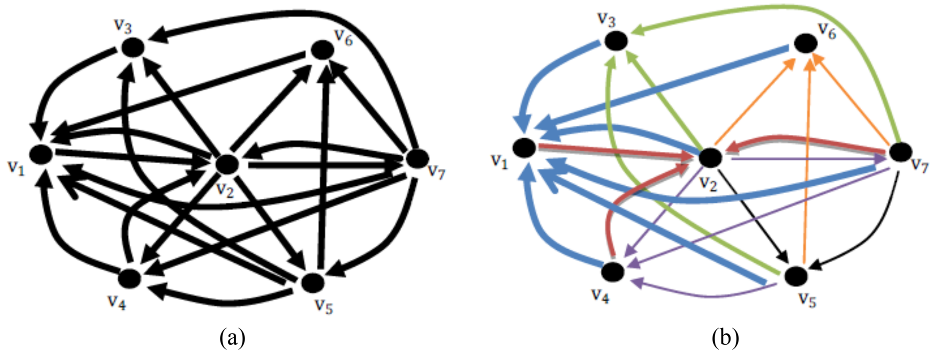

Fig. 4. Graph secondary system of PWR: (a) crisp graph, G_S (b) Fuzzy graph, G_{s_F}

Table 2. Edge descriptions for graph G_S of secondary system PWR

Vertices	Descriptions
(v_3,v_1)	Corrosion occurred in the presence of acid. Iron is dissolved by sulphuric acid to form $FeSO_4$ (Gedeon, 1993) $Fe + H_2SO_4 \rightarrow FeSO_4 + H_2$
(v_6,v_1)	The chemical reaction between chlorine and iron to form $FeCl_3$ as stated in (Castelli, 2009)
(v_2,v_1)	The oxygen in moderator (H_2O) is one of the causes of corrosion (Wu, 1989)
(v_1,v_2)	Ferrous oxide that is highly soluble in an acidic solution is the corrosion product. Gedeon (1993) stated that moderator is formed in the presence of hydrogen and ferrous oxide
(v_7,v_1)	Nitric acid (HNO_3) is formed by chemical reaction between moderator and nitrogen dioxide. Low pH is a corrosion factor (Gedeon, 1993)
(v_4,v_1)	Boric acid $B(OH)_3$ is an acidic compound and one of the factors for corrosion occurrence as stated in (Gedeon, 1993)
(v_5,v_4)	The chemical reaction between sodium hydroxide and boric acid formed sodium borate ($Na_2B_4O_7$)
(v_5,v_3)	The chemical reaction between acid sulphuric with sodium hydroxide formed sodium sulphate (Na_2SO_4)
(v_2,v_3)	The sulfuric acid compound is the products of sulfur trioxide (SO_3) infused with the moderator (Lin, 1996)
(v_4,v_2)	Boric acid heated above 170 °C forms moderator compound as stated in (Lin, 1996)
(v_2,v_4)	When diborane (B_2H_6) is infused with the moderator, the boric acid compound is crystallized (Lin, 1996)
(v_7,v_4)	The chemical reaction between ammonia and boric acid formed Boron Nitride (BN)
$(v_2 v_6)$	Chloride compound is formed based on chemical reaction between hydrochloric acid (HCl) with hydroxide ($OH-$)
(v_2,v_5)	The chemical reactions between moderator and sodium (Na) produced sodium hydroxide (NaOH) compound
(v_5,v_6)	Sodium Chloride (NaCl) compound is formed based on chemical reaction between sodium hydroxide with hydrochloride acid (Lin, 1996)
(v_7,v_6)	Ammonia compound reacted with hydrochloride acid forming ammonium chloride (NH_4Cl)
(v_7,v_3)	The reaction of a sulphuric acid with base ammonia showed that salt $(NH_4)_2SO_4$ is formed (Lin, 1996)
(v_7,v_2)	Nitrogen is used to control pH in the moderator. The chemical reaction between nitrogen with moderator occurs as stated in (Gedeon, 1993)
(v_2,v_7)	The chemical reaction between nitrogen and hydrogen forms NH_3 (Gedeon, 1993); i.e., $3H_2 + N_2 \rightleftarrows 2 NH_3$
(v_7,v_5)	The chemical reaction between with form sodium nitrate ($NaNO_3$).
(v_5,v_1)	The excess sodium are caused caustic Stress Corrosion Cracking (SCC) to the equipment in secondary system (Winters *et al*., 2009b)

Results and Discussion

The dynamic model proposed in (Sabariah *et al*., 2002) is used to model secondary system of PWR. The concentration dynamics of the initial phase for crisp graph and fuzzy graph are given in the following Table 3. The rate of change for corrosion is a positive value for fuzzy graph compared to a negative value for crisp graph. The corrosion value should be positive since the PWR nuclear reactor had been in operation for more than one year before shutting down, which caused the corrosion values to increase.

However, the rate of change for NaOH, Cl_2 and N_2 are negative for fuzzy graph compared to positive values for crisp graph. The concentration of NaOH, Cl_2 and N_2 should have been decreased since it was being consumed during the process. Furthermore, the rate of change values obtained for $B(OH)_3$ from fuzzy graph is less than crisp graph. This is reasonable since the amount of boric acid in the diluted moderator is small (Winters *et al*., 2009a). Table 4 shows the comparison sequence of depletion variables and the variables that remained for crisp graph and fuzzy graph G_S for the PWR system.

Table 3. Concentration dynamics for crisp graph and fuzzy graph

Vertices	Variable	Rate of change for crisp graph G_S	Rate of change for fuzzy graph G_{S_F}
V_1	Corrosion (Co, Zn, Ni, Fe, Zr)	-0.0704	0.1884
V_2	H_2O	-0.3027	-0.0599
V_3	H_2SO_4	-0.2821	-0.0699
V_4	$B(OH)_3$	0.3323	0.0127
V_5	NaOH	0.2991	-0.0043
V_6	Cl_2	0.0136	-0.0373
V_7	N_2	0.0100	-0.0297

Table 4. Sequence of depletion variables for crisp and fuzzy graph

	Crisp graph G_S	Fuzzy graph G_{S_F}
Sequence of variables that are depleted	N_2, NaOH, Cl_2 $B(OH)_3$, H_2SO_4	NaOH, $B(OH)_3$, Cl_2, N_2, H_2SO_4
Variables that left when system is shut down	H_2O and Corrosion (Co, Zn, Ni, Fe, Zr)	H_2O and Corrosion (Co, Zn, Ni, Fe, Zr)

The sequences of depletion for fuzzy graph G_{S_F} are more justifiable than the crisp graph G_S since N_2 should not be depleted as early since there is more N_2 in the secondary system than NaOH. Moreover, $B(OH)_3$ is depleted earlier than Cl_2 since the quantity of boric acid is less than chlorides. However, variables that survived at the end of the operation for both graphs are the same. The corrosion's product and moderators exist till the end of the operation, which confirmed by Murray (2009).

Conclusion

Graph representation of Autocatalytic Set for secondary system of Pressurizer Water Reactor is successfully presented. The procedure and the development of crisp and fuzzy graph are described in the paper. Moreover, the results from fuzzy graph show that the concentration dynamics and the sequence of depletion variables are better than crisp graph.

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Author's Contributions

Azmirul Ashaari and Tahir Ahmad: Contributed in developing the graph of secondary system of PWR.

Mustaffa Shamsuddin: Confirmed the chemical reaction equations.

Wan Munirah Wan Mohamad and Nazira Omar: Provided the literature reviews.

Ethics

The authors declare that there is no conflict of interests regarding the publication of this article.

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