

# Economic Order Quantity with Linearly Time Dependent Demand Rate and Shortages

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**Abstract:** This paper presents an inventory model with linearly time dependent rate and shortages under trade credits. We show that the total cost per unit is convex function of time. Some properties have also been discussed based an optimal solution. The results are discussed with the help of numerical examples. Sensitivity analyses with a variety of numerical results showing the effect of model parameters on key performance measures are demonstrated. Mathematica 7 software is used for finding numerical solutions.

**Keywords:** Inventory, Linearly-Time Dependent Demand, Shortages, Credit

## Introduction

The classical inventory analysis assumes that the supplier is part for the item as soon as the retailer receives the items. But, in real life, the supplier allows a certain (called credit period) to settle the account. During the fixed period, the retailer can start to accumulate revenues on the sales and earn interest on that revenue, but after this period vendor charges interest. The effect of the trade credit on the optimal inventory policy is examined by several researchers like Bregman (1993; Chapman and Ward, 1988; Ward and Chapman, 1987; Daellenbach, 1986; Chapman *et al.*, 1985; Kingsman, 1983; Davis and Gaither, 1985; Haley and Higgins, 1973). Hwang and Shinn (1997) developed the problem of determining the retailer's optimal lot-size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is represented by consultant price elasticity function. Goyal (1985) developed an inventory model under permissible delay in payments. Chung (1988) presented the same model as Goyal (1985) and developed an alternative approach for finding a theorem to determine the economic order quantity order quantity under conditions of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal (1985) model to the case of deterioration. Jamal *et al.* (1997) generalized the Aggarwal and Jaggi (1995) model to the case of allowable shortage. Chung (1989) developed the Discounted Cash Flows (DCF) approach for the analysis of the optimal inventory is the presence of trade credit. The model of Chung (1989) was extended by Jaggi and

Aggarwal (1994) to obtain the optimal order quantity of deteriorating items in the presence of trade credit using DCF approach. Chung *et al.* (2005) developed the problem of determining the economic order quantity under conditions of permissible delay in payments and delay in payments depends on the quantity ordered. The effect of supplier credit policies on optimal order quantity has received the attention of many researchers like Chang and Dye (2001; Chang *et al.*, 2001; Chu *et al.*, 1998; Chen and Chung, 1999; Liao *et al.*, 2000; Arcelus *et al.*, 2003; Abad and Jaggi, 2003; Liao, 2008; Khanna *et al.*, 2011; Liao, 2007; Teng, 2009; Tsao, 2009; Chung and Liao, 2009).

In reality, often some customers are willing to wait until replenishment, especially if the waiting time is short, while others may go elsewhere. Large number of research papers presented by assuming that during stock-out either all demand is backlogged or all is lost. Abad (2001) considered a pricing and lot sizing problem for a product with a variable rate of deterioration allowing shortages and partial backlogging. Dye (2007) amended Abad (2001) model by adding both the backorder cost and the cost of lost sales into the total profit. Dye *et al.* (2007) developed a deterministic inventory model for deteriorating items with price-dependent demand and shortages. Chakraborttya *et al.* (2013) developed a manufacturing inventory model with shortages where carrying cost, shortage cost, set up cost and demand quantity are considered as fuzzy numbers. Janakiraman *et al.* (2013) analyzed the new vendor model and the multi-period inventory model and provided some new results.

Pentico *et al.* (2009) presented the deterministic EPQ with partial backordering: A new approach. Zhang (2012) extended the model of Zhang *et al.* (2011) to make it more applicable to deal with the inventory replenishment problem for multiple associated items. Tripathi (2012) developed an inventory model for exponential time dependent demand rate and shortages.

Recently, Taleizadeh and Nematollahi (2014) investigated the effects of time value of money and inflation on the optimal ordering policy in an inventory control system. Wee *et al.* (2014) proposed an EOQ model with partial backorders considering linear and fixed backordering costs. Ouyang and Chang (2013) studied the optimal production policy for an EPQ inventory system with imperfect production process under permissible delay in payments and complete backlogging. Jaggi *et al.* (2013) developed an EOQ based inventory model for imperfect quantity items to determine the optimal ordering policies of a retailer under permissible delay in payments with allowable shortages. Ghiami *et al.* (2013) investigated a two-echelon supply chain model for deteriorating inventory in which the retailer's warehouse has a limited capacity.

The rest of the paper is organized as follows. In section 2, notations and assumptions are given. Section 3 formulates the model of linearly time dependent demand. In section 4, determination of optimal solution has been given. Section 5 addresses numerical solution followed by sensitivity analysis of different parameters in section 6. Finally concluding remarks and future research is made in the last section 7.

## Notations and Assumptions

The following notations and assumptions are used to develop this manuscript:

- s : per unit shortage cost;
- h : per unit holding cost excluding interest charges; where  $h = h(t) = h \cdot t$ ;
- P : per unit purchase cost;
- A : ordering cost \$/ order
- I(t) : inventory level at time t;
- $I_e$  : annual rate at which interest is earned;
- $I_r$  : annual rate at which interest charged;
- m : permissible delay in settling the account;
- T : length of replenishment cycle;
- $T_1$  : time when inventory level comes down to zero;
- D(t) : demand rate which is  $(a + bt)$ ;
- $Z(T, T_1)$  : average total inventory cost per unit time;

$$Z(T, T_1) = \begin{cases} Z_1(T, T_1), T_1 \geq m \\ Z_2(T, T_1), T_1 < m \end{cases}$$

In addition, the following assumptions are used to develop this proposed model:

- Shortages are allowed and completely backlogged
- The inventory system involves only one item
- Replenishment occurs instantaneously on ordering i.e., lead time is zero
- The demand rate is linearly time dependent and is given by  $(a + bt)$
- No payment to the supplier is outstanding at the time of placing an order i.e.,  $m < T$
- The planning period is of infinite length
- The planning horizon is divided into subintervals of length T units. Orders are placed at time points 0, T, 2T, 3T... The order quantity at each reorders point being just sufficient to bring the stock height to a certain maximum level

## Mathematical Formulation

The inventory level I (t) at time 't' generally decreases mainly to meet the demand only. Thus the variation of inventory with respect to time can be described by the following differential Equation:

$$\frac{dI(t)}{dt} = -(a + bt) \quad (1)$$

$$\text{With the boundary conditions } I(0) = Q, I(T_1) = 0 \quad (2)$$

Solution of Equation 1 using Equation 2, we obtain:

$$I(t) = a(T_1 - t) + \frac{b}{2}(T_1^2 - t^2) \quad (3)$$

Using Equation 3, the order quantity is given by Equation 4:

$$Q = T_1 \left( a + \frac{b}{2} T_1 \right) \quad (4)$$

In the interval  $(0, T_1)$ , the holding cost can be calculated as follows:

$$HC = h \int_0^{T_1} t \cdot I(t) dt = \frac{hT_1^3}{2} \left( \frac{a}{3} + \frac{bT_1}{4} \right) \quad (5)$$

The shortage cost SC over the time interval  $(T_1, T)$  is given by:

$$SC = s \int_{T_1}^T -I(t) dt = \frac{s(T - T_1)}{2} \left\{ a(T - T_1) + \frac{b}{3}(T^2 - 2T_1^2 + TT_1) \right\} \quad (6)$$

Regarding interest payable and interest earned, the following two cases arise based on the values of  $T_1$  and m.

**Case I.  $m \leq T_1$**

Since the length of period with positive stock is larger than the credit period, the buyer can use the sales revenue to earn interest at the annual rate  $I_e$  in  $(0, T_1)$ . The interest earned  $IE_1$  is given by:

$$IE_1 = pI_e \int_0^{T_1} I(t)dt = pI_e \left( \frac{a}{2} + \frac{bT_1}{3} \right) T_1^2 \tag{7}$$

Beyond the credit period, the unsold stock is assumed to be financed with an annual rate  $I_r$  and the interest payable  $IC_1$  is given by:

$$IC_1 = pI_r \int_m^{T_1} I(t)dt = \frac{pI_r(T_1 - m)}{2} \left\{ a(T_1 - m) + \frac{b}{3} \right\} \left( 2T_1^2 - m^2 + mT_1 \right) \tag{8}$$

Thus the total average cost per unit time is given by Equation 9:

$$Z_1(T, T_1) = \frac{A + HC + SC + IC_1 - IE_1}{T} \tag{9}$$

Putting values of  $HC, SC, IC_1, IE_1$  from Equation 5-8 and simplifying, we get:

$$Z_1(T, T_1) = \frac{bs}{6}(T^2 - 3T_1^2) + \frac{as}{2}(T - 2T_1) + \frac{1}{T} \left\{ \frac{bh}{8}T_1^4 + \frac{1}{6}(ah + 2b(s + p(I_r - I_e)))T_1^3 + \frac{a}{2}(s + p(I_r - I_e))T_1^2 - \left( T_1^2 - \frac{apI_r}{2}m(2T_1 - m) + \frac{bpI_r}{6}m(m^2 - T_1^2 - 2mT_1) + \frac{apI_r}{2}m^2 + \frac{bpI_r}{6}m^3 + A \right) \right\} \tag{10}$$

**Case 2.  $M > T_1$**

In this case, the buyer pays no interest but earns interest at an annual rate  $I_e$  during the period  $(0, m)$ . Interest earned  $IE_2$  in this case is given by:

$$IE_2 = pI_e \left\{ \int_0^{T_1} t.(a + bt)dt + (m - T_1) \int_0^{T_1} (a + bt)dt \right\} = \frac{pI_e T_1}{2} \left\{ a(2m - T_1) + bT_1 \left( m - \frac{T_1}{3} \right) \right\} \tag{11}$$

Therefore the total average cost per unit time is given by:

$$Z_2(T, T_1) = \frac{A + HC + SC - IE_2}{T} \tag{12}$$

Putting values of  $HC, SC, IE_2$  from Equation 5, 11 and 12 and simplifying, we get:

$$Z_2(T, T_1) = \frac{bs}{6}(T^2 - 3T_1^2) + \frac{as}{2}(T - 2T_1) + \frac{1}{T} \left\{ \frac{bh}{8}T_1^4 + \frac{1}{6}(ah + 2b(s + pI_e))T_1^3 + \frac{a}{2}(s + pI_e)T_1^2 - \left( apI_e m T_1 - \frac{bpI_e}{2} m T_1^2 + A \right) \right\} \tag{13}$$

**Determination of Optimal Solution**

To find the optimal solution for the problem, we minimize  $Z_i(T, T_1)$  for Case I and Case II respectively and then compare them to obtain minimum value. Our aim is to find minimum average cost per time unit for both cases i.e., Case I and II respectively with respect to  $T$  and  $T_1$ . The necessary and sufficient condition to minimise  $Z_i(T, T_1)$ ;  $i = 1, 2$  for given values of  $T$  are respectively

$$\frac{\partial Z_1}{\partial T} = 0, \frac{\partial Z_1}{\partial T_1} = 0, \frac{\partial Z_2}{\partial T} = 0, \frac{\partial Z_2}{\partial T_1} = 0 \quad \text{and}$$

$\frac{\partial^2 Z_1}{\partial T^2} > 0, \frac{\partial^2 Z_2}{\partial T^2} > 0$ . Differentiating Equation 10 and 13 partially with respect to  $T$  and  $T_1$  and two times, we get Equation 14-22:

$$\frac{\partial Z_1}{\partial T} = 6asT + 6bsTT_1 - 3bhT_1^3 - 3(ah + 2b(s + p(I_r - I_e)))T_1^2 - 6a(s + p(I_r - I_e))T_1 + 2pI_r(3a + b)m + 2bpI_r m T_1 \tag{14}$$

$$\frac{\partial Z_1}{\partial T_1} = 8bsT^3 + 12asT^2 - 3bhT_1^4 - 4(ah + 2b(s + p(I_r - I_e)))T_1^3 - 12a(s + p(I_r - I_e))T_1^2 + 12apI_r m(2T_1 - m) - 4bI_r m(m^2 - T_1^2 - 2mT_1) - 12apI_r m^2 - 4bpI_r m^3 - 24A \tag{15}$$

$$\frac{\partial Z_2}{\partial T} = 2asT + 2bsTT_1 - bhT_1^3 - (ah + 2b(s + pI_e))T_1^2 - 2a(s + pI_e)T_1 + 2apI_e m + 2bpI_e m T_1 \tag{16}$$

$$\frac{\partial Z_2}{\partial T} = 8bsT^3 + 12asT^2 - 3bhT_1^4 - 4(ah + 2b(s + pI_e))T_1^3 - 12a(s + pI_e)T_1^2 + 24apI_e m T_1 - 12apI_e m T_1 - 24A \tag{17}$$

$$\frac{\partial^2 Z_1}{\partial T^2} = \frac{bs}{3} + \frac{2}{T^3} \left\{ \frac{bh}{8}T_1^4 + \frac{1}{6}(ah + 2b(s + p(I_r - I_e)))T_1^3 + \frac{a}{2}(s + p(I_r - I_e))T_1^2 + \frac{a}{2}(s + p(I_r - I_e))T_1^2 - \frac{apI_r}{2}m(2T_1 - m) + \frac{bpI_r}{6}m(m^2 - T_1^2 - 2mT_1) + \frac{apI_r}{2}m^2 + \frac{apI_r}{2}m^2 + \frac{bpI_r}{6}m^3 + A \right\} > 0 \tag{18}$$

$$\frac{\partial^2 Z_1}{\partial T_1^2} = \frac{1}{T} \left\{ \frac{3bh}{2}T_1^2 + (ah + 2b(s + p(I_r - I_e)))T_1 + a(s + p(I_r - I_e)) + \frac{bpI_r}{3}m \right\} - bs > 0 \tag{19}$$

$$\frac{\partial^2 Z_2}{\partial T^2} = \frac{bs}{3} + \frac{2}{T^3} \left\{ \frac{bh}{8} T_1^4 + \frac{1}{6} (ah + 2b(s + pI_e)) T_1^3 \right. \\ \left. + \frac{a}{2} (s + pI_e) T_1^2 + \frac{a}{2} (s + pI_e) T_1^2 - apI_e m T_1 \right. \\ \left. - \frac{bpI_e}{2} m T_1^2 + A \right\} > 0 \quad (20)$$

$$\frac{\partial^2 Z_2}{\partial T_1^2} = \frac{1}{T} \left\{ \frac{3bh}{2} T_1^2 + (ah + 2b(s + pI_e)) T_1 + \right. \\ \left. a(s + pI_e) T_1 - bpmI_e \right\} - bs > 0 \quad (21)$$

$$\frac{\partial^2 Z_1}{\partial T^2} = \frac{bs}{3} + \frac{2}{T^3} \left\{ \frac{bh}{8} T_1^4 + \frac{1}{6} (ah + 2b(s + pI_e)) T_1^3 + \right. \\ \left. \frac{a}{2} (s + pI_e) T_1^2 - apI_e m T_1 - \frac{bpI_e}{2} m T_1^2 + A \right\} > 0 \quad (22)$$

For finding optimal (minimum) values of  $T_1 = T_1^*$ ,  $T = T^*$  for case I and  $T_1 = T_1^{**}$ ,  $T = T^{**}$  for case II is obtained by

solving  $\frac{\partial Z_1}{\partial T_1} = 0$ ,  $\frac{\partial Z_2}{\partial T} = 0$ ; and  $\frac{\partial Z_1}{\partial T_1} = 0$ ,  $\frac{\partial Z_2}{\partial T} = 0$ , we get Equation 23 and 24:

$$\begin{cases} 6asT + 6bsTT_1 - 3bhT_1^3 - 3(ah + 2b(s + p(I_r - I_e)))T_1^2 \\ -6a(s + p(I_r - I_e))T_1 + 2pI_e(3a + b)m + 2bpI_e m T_1 = 0, \\ 8bsT^3 + 12asT^2 - 3bhT_1^4 - 4(ah + 2b(s + p(I_r - I_e)))T_1^3 \\ -12a(s + p(I_r - I_e))T_1^2 + 12apI_e m(2T_1 - m) \\ -4bI_e m(m^2 - T_1^2 - 2mT_1) - 12apI_e m^2 - 4bpI_e m^3 - 24A = 0 \end{cases} \quad (23)$$

$$\begin{cases} 2asT + 2bsTT_1 - bhT_1^3 - (ah + 2b(s + pI_e))T_1^2 \\ -2a(s + pI_e)T_1 + 2apI_e m + 2bpI_e m T_1 = 0, \\ 8bsT^3 + 12asT^2 - 3bhT_1^4 - 4(ah + 2b(s + pI_e))T_1^3 - \\ 12a(s + pI_e)T_1^2 + 24apI_e m T_1 - 12apI_e m T_1 - 24A = 0 \end{cases} \quad (24)$$

## Numerical Examples

### Example 1. Case I

Let A = \$100 per order, h = \$30 per unit, p = \$100 per unit, s = \$50 per unit, a = 3600, b = 2400,  $I_e = 0.1$ ,  $I_r = 0.2$ , m = 90/365 year. Optimal replenishment cycle time  $T = T^* = 1.5323$  year, optimal value of  $T_1 = T_1^* = 1.32973$  year, optimal total inventory cost  $Z_1(T, T_1) = Z_1^*(T^*, T_1^*) = \$ 68236.5$  and optimal order quantity  $Q = Q_1^* = 6908.85$ .

### Example 2. Case II

Let A = \$100 per order, h = \$30 per unit, p = \$100 per unit, s = \$50 per unit, a = 3600, b = 2400,  $I_e = 0.1$ ,  $I_r = 0.2$ , m = 90/365 year. Optimal replenishment cycle time  $T = T^* = 0.125372$  year, optimal value of  $T_1 = T_1^* = 0.123279$  year, optimal total inventory cost  $Z_2(T, T_1) = Z_2^*(T^*, T_1^*) = \$ 408.042$  and optimal order quantity  $Q = Q_2^* = 462.042$ .

## Sensitivity Analysis

### Case I

Table 1. Variation of 'h' keeping all parameters same as in given example 1

| h  | T*       | T <sub>1</sub> * | Q <sub>1</sub> * | Z <sub>1</sub> *(T*, T <sub>1</sub> *) |
|----|----------|------------------|------------------|--|
| 10 | 4.782830 | 4.126720         | 35292.0          | 461085.0                               |
| 20 | 1.423740 | 1.275280         | 6542.61          | 27397.60                               |
| 40 | 0.604120 | 0.556229         | 2373.69          | 10264.30                               |
| 50 | 0.527716 | 0.483857         | 2022.83          | 9032.040                               |
| 60 | 0.481613 | 0.438680         | 1810.18          | 8687.420                               |

Table 2. Variation of 'A' keeping all parameters same as in given example 1

| A   | T*       | T <sub>1</sub> * | Q <sub>1</sub> * | Z <sub>1</sub> *(T*, T <sub>1</sub> *) |
|-----|----------|------------------|------------------|--|
| 50  | 0.766405 | 0.704292         | 3130.68          | 14667.4                                |
| 80  | 0.767665 | 0.705239         | 3135.69          | 14757.7                                |
| 120 | 0.769333 | 0.706496         | 3142.35          | 14877.6                                |
| 150 | 0.770578 | 0.707434         | 3147.32          | 14967.2                                |
| 200 | 0.772641 | 0.708986         | 3155.54          | 15116.0                                |

Table 3. Variation of 'p' keeping all parameters same as in given example 1

| p   | T*       | T <sub>1</sub> * | Q <sub>1</sub> * | Z <sub>1</sub> *(T*, T <sub>1</sub> *) |
|-----|----------|------------------|------------------|--|
| 50  | 0.806583 | 0.746813         | 3357.80          | 15292.4                                |
| 80  | 0.779490 | 0.718381         | 3205.46          | 14873.1                                |
| 120 | 0.760915 | 0.696464         | 3089.34          | 14868.8                                |
| 150 | 0.753629 | 0.686095         | 3034.81          | 15071.7                                |
| 200 | 0.747787 | 0.674607         | 2974.71          | 15600.9                                |

Table 4. Variation of 'm' keeping all parameters same as in given example 1

| m       | T*       | T <sub>1</sub> * | Q <sub>1</sub> * | Z <sub>1</sub> *(T*, T <sub>1</sub> *) |
|---------|----------|------------------|------------------|--|
| 50/365  | 0.462845 | 0.434012         | 1923.31          | 6065.29                                |
| 60/365  | 0.550096 | 0.512569         | 2343.47          | 8190.76                                |
| 70/365  | 0.622977 | 0.583609         | 2743.10          | 10397.20                               |
| 80/365  | 0.702241 | 0.647698         | 3119.84          | 12615.40                               |
| 100/365 | 0.829567 | 0.759722         | 3812.26          | 16995.90                               |

### Case II

Table 5. Variation of 'h' keeping all parameters same as in given example 2

| h  | T**      | T <sub>1</sub> ** | Q <sub>2</sub> ** | Z <sub>2</sub> *(T*, T <sub>1</sub> *) |
|----|----------|-------------------|-------------------|--|
| 10 | 0.160713 | 0.159891          | 606.286           | 163.932                                |
| 20 | 0.138219 | 0.136680          | 514.466           | 302.287                                |
| 40 | 0.116640 | 0.114084          | 426.321           | 495.512                                |
| 50 | 0.110152 | 0.107194          | 399.687           | 570.954                                |
| 60 | 0.105059 | 0.101744          | 378.701           | 637.737                                |

Table 6. Variation of 'A' keeping all parameters same as in given example 2

| A   | T**      | T <sub>1</sub> ** | Q <sub>2</sub> ** | Z <sub>2</sub> *(T*, T <sub>1</sub> *) |
|-----|----------|-------------------|-------------------|--|
| 80  | 0.115261 | 0.114013          | 426.046           | 241.886                                |
| 120 | 0.134193 | 0.131325          | 493.466           | 562.092                                |
| 150 | 0.145727 | 0.141792          | 534.577           | 776.338                                |
| 200 | 0.161896 | 0.156365          | 581.470           | 1101.16                                |

Table 7. Variation of ‘p’ keeping all parameters same as in given example 2

| p  | $T^{**}$ | $T_1^{**}$ | $Q_2^{**}$ | $Z_2^*(T^*, T_1^*)$ |
|----|----------|------------|------------|---------------------|
| 2  | 0.137056 | 0.132265   | 515.943    | 939.799             |
| 4  | 0.134050 | 0.129919   | 504.143    | 808.901             |
| 6  | 0.131105 | 0.127644   | 492.604    | 676.656             |
| 8  | 0.128214 | 0.125433   | 481.297    | 543.043             |
| 15 | 0.118437 | 0.118107   | 441.924    | 64.2153             |

Table 8. Variation of ‘m’ keeping all parameters same as in given example 2

| m       | $T^{**}$ | $T_1^{**}$ | $Q_2^{**}$ | $Z_2^*(T^*, T_1^*)$ |
|---------|----------|------------|------------|---------------------|
| 50/365  | 0.125970 | 0.121821   | 456.364    | 808.475             |
| 60/365  | 0.125858 | 0.122222   | 457.917    | 709.007             |
| 70/365  | 0.125722 | 0.122596   | 459.381    | 609.112             |
| 80/365  | 0.125560 | 0.122949   | 460.756    | 508.789             |
| 100/365 | 0.125159 | 0.123585   | 463.234    | 306.851             |

All the above observations from Table 1 to 8 sum up as follows:

**Case I**

- From Table 1: It is observed that increase of holding cost ‘h’ results decrease in optimal cycle time  $T = T^*$ ,  $T = T_1^*$ , optimal order quantity  $Q = Q_1^*$  and total relevant cost  $Z_1(T^*, T_1^*)$ . That is, change in holding cost leads negative change in  $T = T^*$ ,  $T = T_1^*$ ,  $Q = Q_1^*$  and  $Z_1(T^*, T_1^*)$
- From Table 2: Increase of ordering cost ‘A’ results slight increase in optimal cycle time  $T = T^*$ , value of  $T_1^*$ , optimal order quantity  $Q = Q_1^*$  and total relevant cost  $Z_1(T^*, T_1^*)$ . That is, change in ordering cost leads slight positive change in  $T = T^*$ ,  $T = T_1^*$ ,  $Q = Q_1^*$  and positive change in  $Z_1(T^*, T_1^*)$
- From Table 3: Increase of purchase cost ‘p’ results decrease in optimal cycle time  $T = T^*$ , value of  $T_1^*$ , optimal order quantity  $Q = Q_1^*$  and total relevant cost  $Z_1(T^*, T_1^*)$ . That is, change in purchase cost leads negative change in  $T = T^*$ ,  $T = T_1^*$ ,  $Q = Q_1^*$  and  $Z_1(T^*, T_1^*)$
- From Table 4: Increase of credit period ‘m’ results increase in optimal cycle time  $T = T^*$ , value of  $T_1^*$ , optimal order quantity  $Q = Q_1^*$  and total relevant cost  $Z_1(T^*, T_1^*)$ . That is change in credit period leads positive change in  $T = T^*$ ,  $T = T_1^*$ ,  $Q = Q_1^*$  and  $Z_1(T^*, T_1^*)$

**Case II**

- From Table 5: Increase of holding cost ‘h’ results decrease in optimal cycle time  $T = T^{**}$ , value of  $T_1^{**}$ , optimal order quantity  $Q = Q_2^{**}$  and increase in total relevant cost  $Z_2(T^{**}, T_1^{**})$ . That is, change in holding cost leads negative change in  $T = T^{**}$ ,  $T = T_1^{**}$ ,  $Q = Q_1^{**}$  and  $Z_2(T^{**}, T_1^{**})$
- From Table 6: Increase of ordering cost ‘A’ results increase in optimal cycle time  $T = T^{**}$  value of  $T_1^{**}$ ,

optimal order quantity  $Q = Q_2^{**}$  and total relevant cost  $Z_2(T^{**}, T_1^{**})$ . That is, change in ordering cost leads positive change in  $T = T^{**}$ ,  $T = T_1^{**}$ ,  $Q = Q_1^{**}$  and positive change in  $Z_2(T^{**}, T_1^{**})$

- From Table 7: Increase of purchase cost ‘p’ results decrease in optimal cycle time  $T = T^{**}$ , value of  $T_1^{**}$ , optimal order quantity  $Q = Q_2^{**}$  and total relevant cost  $Z_2(T^{**}, T_1^{**})$ . That is, change in purchase cost leads negative change in  $T = T^{**}$ ,  $T = T_1^{**}$ ,  $Q = Q_1^{**}$  and  $Z_1(T^*, T_1^*)$
- From Table 8: Increase of credit period ‘m’ results slight decrease in optimal cycle time  $T = T^{**}$ , value of  $T_1^{**}$ , optimal order quantity  $Q = Q_2^{**}$  and decrease in total relevant cost  $Z_2(T^{**}, T_1^{**})$ . That is change in credit period leads slight negative change in  $T = T^{**}$ ,  $T = T_1^{**}$ ,  $Q = Q_1^{**}$  and negative change in  $Z_2(T^{**}, T_1^{**})$

**Conclusion and Future Research**

This study develops an inventory model for a linear time-dependent demand rate, where holding cost is proportional to time, when a supplier provides a permissible delay in payments. In this study, an optimal procedure is presented to obtain optimal replenishment cycle time, optimal average total cost with the optimal order quantity. Numerical examples are given to illustrate the proposed model. From managerial point of view the following observation is made: (i) increase of holding cost results decrease in total cost for case I and increase of total cost for case II (ii) increase of ordering cost results increase of total cost (iii) increase of purchase cost results decrease of total cost (iv) increase of credit period results increase of total cost for case I and decrease of total cost for case II.

The model proposed in this study can be extended in several ways. For instance, extension could include deterioration rate and demand rate as a function of quantity as well as quadratic time variation could be considered. Finally the model can be generalized with stochastic market demand when a supplier provides a permissible delay in payments and a cash discount.

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**Author’s Contributions**

**R.P. Tripathi:** Paper formation, Mathematical formulation, discussion of data-

analysis, contributed to the writing of the manuscript and publication of the manuscript.

**D. Singh:** Coordination the mouse of work and publication of the manuscript.

**Tushita Mishra:** Design the research plan, organization, development and publication of this manuscript.

## Ethics

In this paper Truncated Taylor's series method have been used for exponential terms to find closed form optimal solution. With the help of differential calculus the author's obtained the minimum total average cost per unit time.

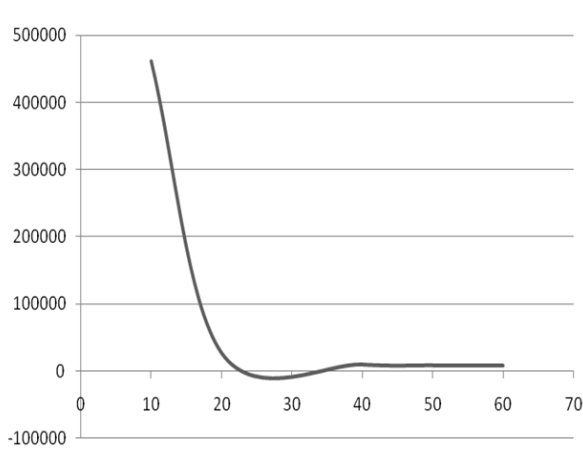
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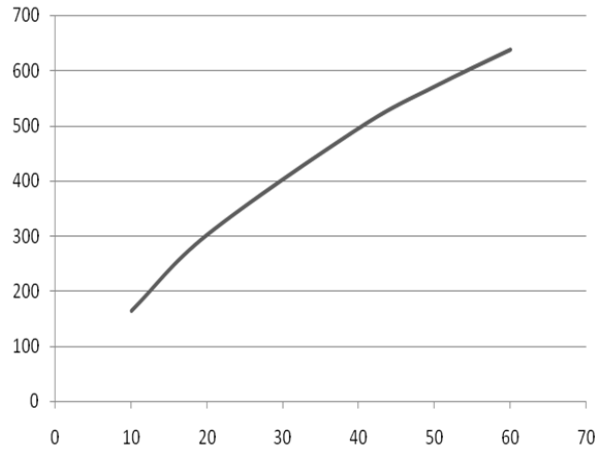
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## Appendix

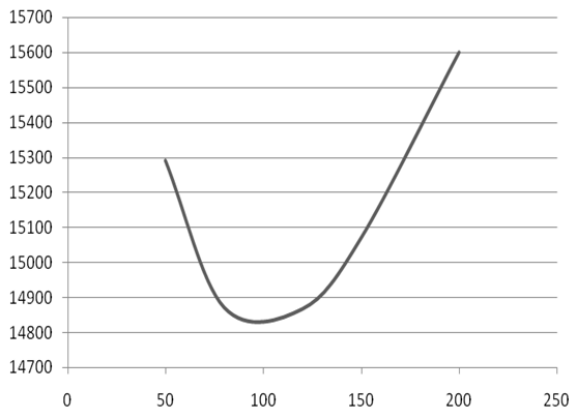
The figures are given to clarify the sensitivity analysis with respect to the parameters ‘h’ ‘p’ and ‘A’ with total relevant cost for both cases:



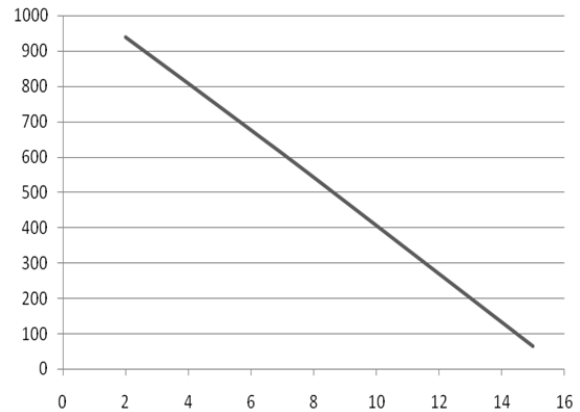
Case I: Graph  $h$  Vs  $Z_1(T)$



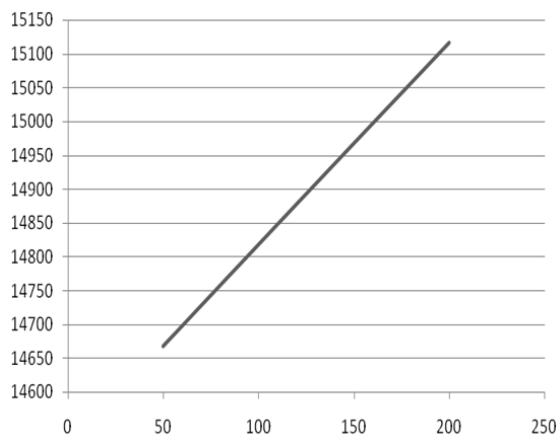
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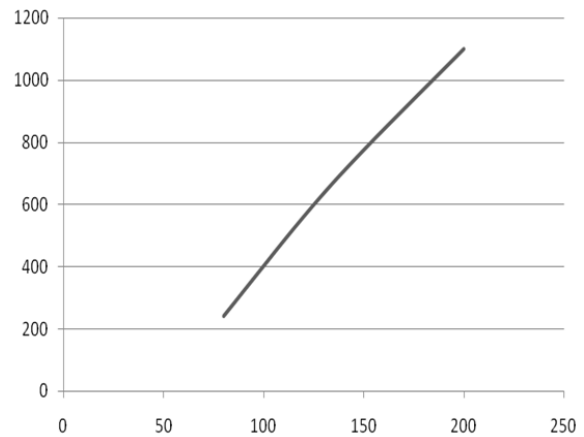
Case I: Graph  $p$  Vs  $Z_1(T)$



Case I: Graph  $p$  Vs  $Z_2(T)$



Case I: Graph ‘A’ Vs  $Z_1(T)$



Case I: Graph ‘A’ Vs  $Z_2(T)$