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Advanced Multivariate Time Series Forecasting Models

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Abstract: In this study, the focus is to collect and summarize various advanced forecasting models for multivariate time series dataset. We have discussed about the inherent forecasting strengths and weaknesses related to these time series modelings. Also, the main section deal with the experience of using such data in econometric analysis. Besides, the implementation of SAS and R softwares improve the parameter estimation and forecasting accuracy. Eventually, we evaluated these forecasting models by different criterions and select the best one for the future tendency of land market value.

Keywords: Advanced Forecasting Model, Macroeconomic Variables, Multivariate Time Series Analysis

Introduction

Nowadays, there are a lot of methods and techniques to analyze and forecasting time series dataset. A lot of researchers have been studying time series forecasting for approximately one century in order to get better forecasts for the future. To achieve best forecast accuracy level, various time series forecasting approaches have been proposed in the literature. After 1980s, more sophisticated algorithms could be improved since properties of computers were enhanced.

This research presents a time series estimation and prediction methods with the use of classic and advanced forecasting tools. Our discussion about different time series models is supported by giving the experimental forecast results, performed on several macroeconomic variables. Also, this paper gives an introduction to the basic concepts of time series modeling, together with some associated ideas such as stationarity and parsimony. Finally, the summary of various existing forecasting models can provide information to develop an appropriate forecasting model which describes the inherent feature of the series.

The forecasting techniques can be broadly categorized as consisting of qualitative and quantitative methods.

Qualitative forecasting techniques are mainly exploratory research. Qualitative forecasts are often used in providing the insights into the problems. However, although some data analysis may be performed, the expectations are based on the mathematical studies within the field of biological mathematics, physical and chemical mathematics and others. Perhaps the Delphi Method is the most formal and widely known qualitative forecasting method. This technique was developed by the RAND Corporation. It employs a panel of experts who are assumed to be knowledgeable about the problem. The panel members are physically separated to avoid their deliberations being impacted either by social pressures or by a single dominant individual.

Quantitative forecasting techniques is used to quantify the problem by way of generating numerical data or data that can be transformed into usable statistics. Quantitative methods are much more structured and reliable results than Qualitative methods. In subsequent sections, as Fig. 1 shown, we will discuss all several different types of quantitative forecasting models.

Advanced Forecasting Models

Smoothing Model

For this method, we will present mechanics of the most important exponential smoothing methods and their application in forecasting time series with various characteristics. This helps us develop an intuition to how these methods work. We apply the simple exponential smoothing method, Holt's linear method, exponential smoothing method and additive damped method and multiplicative damped method. For Exponential Smoothing Methods (ETS) model, we

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take into innovation by considering multiplicative error equations:

$$y_t = (l_{t-1} + b_{t-1})(1 + \varepsilon_t) \tag{1}$$

$$l_t = (l_{t-1} + b_{t-1})(1 - \alpha\varepsilon_t) \tag{2}$$

$$b_t = b_{t-1} + \beta(l_{t-1} + b_{t-1})\varepsilon_t \tag{3}$$

$$\varepsilon_t = \frac{y_t - (l_{t-1} + b_{t-1})}{l_{t-1} + b_{t-1}} \tag{4}$$

where, the $\varepsilon \sim NID(0, \sigma^2)$, l_t denotes an estimate of the level of the series at time t, b_t denotes an estimate of the trend (slope) of the series at time t, α denotes the smoothing parameter for level. β denotes the smoothing parameter for the trend.

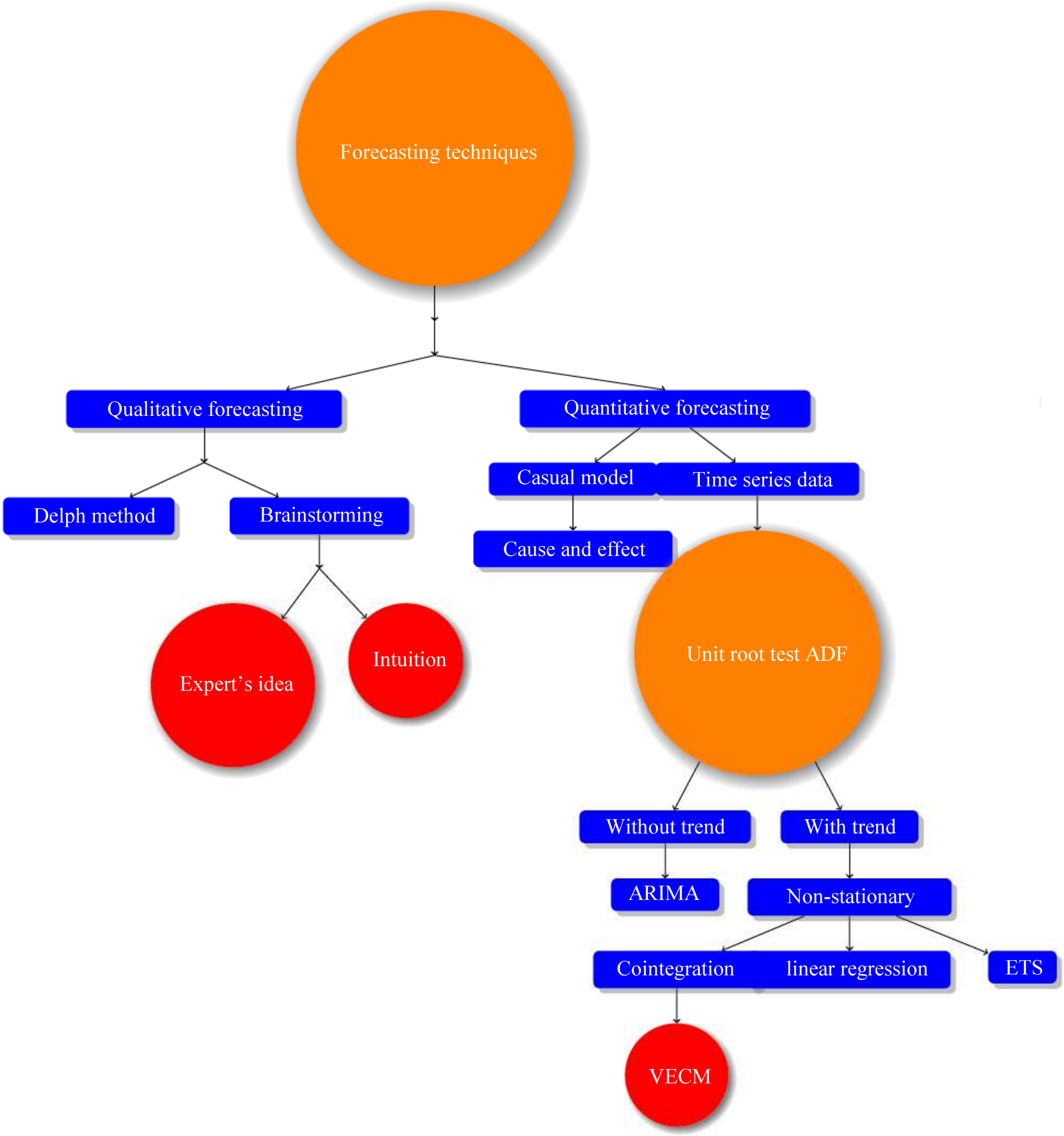

Fig. 1: Summary of advanced forecasting models

Autoregressive Integrated Moving Average Model

The time series processes we have discussed so far are all stationary processes, but many applied time series, particularly those arising from economic and business areas, are non-stationary. With respect to the class of covariance stationary processes, non-stationary time series can occur in many different ways. They could have non-constant means μ_t, time-varying second moments such as non-constant variance σ_t^2, or both of these properties. In this section, we will explain the construction of a very useful class of homogeneous non-stationary time series models, the autoregressive integrated moving average models. Some useful differencing and variance stabilizing transformations are introduced to connect the stationary and non-stationary time series models.

Many models used in practice are of the simple ARIMA type, which have a long history and were formalized in Box and Jenkins (1970). ARIMA stands for Autoregressive Integrated Moving Average and an *ARIMA*(*p*, *d*, *q*) model for an observed series $\{y_t\}$, $t = 1 \cdots T$ is a model where the *dth* difference $z_t = y_t - y_{t-d}$ is taken to induce stationarity of the series. The process $\{z_t\}$ is then modeled as $z_t = \mu + \epsilon_t$ with:

$$\epsilon_t = \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_p \epsilon_{t-p} + u_t - \eta_1 u_{t-1} - \cdots - \eta_q u_{t-q} \quad (5)$$

or in terms of polynomials in the lag operator L (defined through $L^s x_t = x_{t-s}$):

$$\phi(L)\epsilon_t = \eta(L) u_t \quad (6)$$

where, u_t is white noise and usually Normally distributed as $u_t \sim N(0, \sigma^2)$. The stationarity and invertibility conditions are simply that the roots of $\phi(L)$ and $\eta(L)$, respectively, are outside the unit circle circle (Mark, 1980).

Vector Auto-Regression Model

There are a variety of methods available for forecasting economic variables. One common type of forecast is Vector Auto-regression modeling for multivariate Time Series approach. This type of forecast is predominant in economics and financial analysis.

A Vector Auto-Regression (VAR) model is an useful and flexible approach to describe the dynamic behavior of economic activity and financial time series dataset; that is, a vector of time series. In this system, we consider one equation for one variable as dependent variable with constant and lags. Each variable is assumed to influence with each other in the system, which makes direct interpretation of the estimated coefficients very difficult (See Hyndman and Athanasopoulos, 2014). We write a multi-dimensional VAR(p) as:

$$\begin{aligned} Y_t &= C + \Phi_1 \begin{bmatrix} LLMV_{t-1} \\ LCPI_{t-1} \\ LUR_{t-1} \\ LPP_{t-1} \\ LCCI_{t-1} \\ LPMI_{t-1} \end{bmatrix} \\ &+ \Phi_2 \begin{bmatrix} LLMV_{t-2} \\ LCPI_{t-2} \\ LUR_{t-2} \\ LPP_{t-2} \\ LCCI_{t-2} \\ LPMI_{t-2} \end{bmatrix} + \cdots + \Phi_{t-p} \begin{bmatrix} LLMV_{t-p} \\ LCPI_{t-p} \\ LUR_{t-p} \\ LPP_{t-p} \\ LCCI_{t-p} \\ LPMI_{t-p} \end{bmatrix} + a_t \end{aligned} \quad (7)$$

where, at are white noise process. $E(a_t) = 0$ and:

$$E\left(a_t a_t'\right) = \begin{cases} 0 & when\ t = \tau, \\ \Omega & when\ t \neq \tau,. \end{cases}$$

In the reduced form, we will include a six variable VAR with one lag in our forecasting model:

$$Y_t = \begin{bmatrix} LLMV_t \\ LCPI_t \\ LUI_t \\ LCCI_t \\ LPP_t \\ LPMI_t \end{bmatrix} \Phi_1 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & \phi_{45} & \phi_{46} \\ \phi_{51} & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} & \phi_{56} \\ \phi_{61} & \phi_{62} & \phi_{63} & \phi_{64} & \phi_{65} & \phi_{66} \end{bmatrix}$$

Coefficient $\phi_{ii,i}$ indicates the influence of the *ii*th lag of variable Y_i on itself, while coefficient $\phi_{ij,i}$ indicates the influence of the *ii*th lag of variable Y_j on Y_i.

A "VAR in levels" is known as the series modeled are stationary, we forecast them directly by fitting a VAR to the data. A "VAR in differences" is known as the series are non-stationary, we firstly take differences to make them stationary and then we fit a VAR model. In both cases, the models and coefficients are estimated equation by equation using the principle of least squares.

We applied the VAR selection package for forecasting the raw data. The function returns information criteria and final prediction error for sequential increasing the lag order up to a VAR(p)-process. which are based on the same sample size. For each equation, the parameters are estimated by minimizing the sum of squared $e_{i,t}$ values.

Before we ran the *R* software, we take the log transformation of the raw data to stabilize the variance. And then, we set the 80% of the data as training set and the remaining data as the test set (Brockwell and Davis, 2002).

VAR model generate the forecasting in a recursive structure. The VAR system require each variable is regressed on a constant and p las of its own lags as well as on p lags of each of the other variables. To illustrate the process, assume that we have fitted the multi-dimensional VAR (1) described in equations Equation (7) for all observations up to time T.

Then the one-step-ahead forecasts are generated by:

$$\hat{y}_{1,T+1|T} = \hat{c}_1 + \hat{\phi}_{11,1y_{1,T}} + \hat{\phi}_{12,1y_{1,T}} \tag{8}$$

$$\hat{y}_{2,T+1|T} = \hat{c}_1 + \hat{\phi}_{21,1y_{1,T}} + \hat{\phi}_{22,1y_{2,T}} \tag{9}$$

$$\hat{y}_{3,T+1|T} = \hat{c}_1 + \hat{\phi}_{31,1y_{1,T}} + \hat{\phi}_{32,1y_{3,T}} \tag{10}$$

$$\hat{y}_{4,T+1|T} = \hat{c}_1 + \hat{\phi}_{41,1y_{1,T}} + \hat{\phi}_{42,1y_{4,T}} \tag{11}$$

$$\hat{y}_{5,T+1|T} = \hat{c}_1 + \hat{\phi}_{51,1y_{1,T}} + \hat{\phi}_{52,1y_{5,T}} \tag{12}$$

$$\hat{y}_{6,T+1|T} = \hat{c}_1 + \hat{\phi}_{61,1y_{1,T}} + \hat{\phi}_{62,1y_{6,T}} \tag{13}$$

This is the same form as Equation (8) to Equation (13) except that the errors have been set to zero and parameters have been replaced with their estimates.

AR-GARCH Model

In 1982, Engle introduced the Auto-Regressive Conditional Heteroscedasticity (ARCH) and explicitly recognizes this type of temporal dependence. Moreover, Crawford and Fratantoni (2003) applied a Markov-switching model to U.S. home price and compare the performance with ARIMA and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models. In 1991, Nelson put forward Exponential Garch (E-Garch) and Glosten *et al*. (1993), introduced GJR-GARCH model (See Engle, 2001). Besides, Miles (2008) evaluate the Forecasting Performance of Linear and Non-linear Models of House Prices. To correlate changes in volatility with changes in returns, Engle *et al*. (1987) proposed the GARCH-M model (Miles, 2008).

The ARCH model assumes that the changes in variance is a function of the realizations of squares of preceding errors.

To model a time series ϵ_t using an ARCH process, we assume that:

$$\sqrt{h_t}\, Z_t = \epsilon_t \tag{14}$$

$$Var\left(\epsilon_t \mid past\, data\right) = h_t\,. \tag{15}$$

$$\frac{\epsilon_t}{\sqrt{h_t}} \sim IID\left(0,1\right). \tag{16}$$

where, h_t is the conditional variance.

Especially, the order of the model needs to be specified for each of the parametric models before fitting the model to the in-sample data.

The ability to model volatility clustering can be seen in the definition of the conditional variance where it is evident that a large Z_{t-i}^2 will give rise to a large h_t^2 :

$$h_t = \zeta_0 + \zeta_1 Z_{t-1}^2 + \zeta_2 Z_{t-2} + \cdots + \zeta_p Z_{t-p} \tag{17}$$

Hence the conditional variance of Z_t is:

$$Var\left(Z_t \mid Z_{t-1}, \cdots\right) = E\left(Z_t^2 \mid Z_{t-1}^2, \cdots\right) \tag{18}$$

$$= h\left(t\right) \tag{19}$$

$$= \zeta_0 + \zeta_1 Z_{t-1}^2 + \zeta_2 Z_{t-2} + \cdots + \zeta_p Z_{t-p} \tag{20}$$

We can also state that the current conditional variance should also depend on the previous conditional variances as:

$$\begin{aligned} h_t &= \zeta_0 + \varsigma_1 h_{t-1} + \varsigma_2 h_{t-2} + \cdots + \varsigma_p h_{t-p} \\ &+ \zeta_1 Z_{t-1}^2 + \zeta_2 Z_{t-2}^2 + \cdots + \zeta_q Z_{t-q}^2 \end{aligned} \tag{21}$$

In this notation, the error term Z_t follow a generalized autoregressive conditional heteroskedastic process with orders p and q, $GARCH(p, q)$, which is proposed by Bollerslev (2006).

Although the error term, ϵ_t, can be autocorrelated in the regression model, it should be stationary. A non-stationary error structure could produce a spurious regression where a significant regression can be achieved for a totally unrelated series as shown in Clive and Paul (1986) and Peter (1986). In such a case, one should properly difference the series before estimating the regression (Brockwell and Davis, 2002).

To fit the $AR(m)$- $GARCH(p, q)$ model, we consider the following formula:

$$\begin{cases} x_t = f\left(t, x_{t-1}, x_{t-2}, \cdots\right) + \epsilon_t \\ \epsilon_t = \sum_{k=1}^{m} \beta_k \epsilon_{t-k} + v_t \\ v_t = \sqrt{h_t e_t} \\ h_t = \omega + \sum_{i=1}^{p} \eta_i h_{t-i} + \sum_{j=1}^{q} \lambda_j v_{t-j}^2 \end{cases}$$

where, $f(t, x_{t-1}, x_{t-2}, \cdots)$ is the regression function of x_t, $e_t \sim N(0.1)$.

GARCH model is a generalization of ARCH model. In GARCH (p,q), when $p = 0$, it is ARCH(q) model.

To fit a GARCH model for ϵ_t, we must follow.

A portmanteau Q test is used for autocorrelation in errors:

H_0: There is no evidence show that there are autocorrelation in residuals for some lag p.

H_1: There are some evidence show that there are autocorrelation in residuals for some lag p.

The investigation of conditional variance models has been one of the main areas of study in time series analysis of financial markets. Towards these ends, the GARCH model and its variations have been applied to many risk and volatility studies. We use above simple examples to illustrate the procedures (William, 2006).

Bayesian VAR Model

The Bayesian approach was introduced to a reevaluation of the VAR approach based on the Bayesian principles. Thus, the VAR approach was characterized by several deficiencies, especially due to the over-parameterization problems. The Bayesian approach proposes a solution to this problem due to the fact that it does not ponder too much any of the parameters of the model. However the emphasis falls on the use of prior distributions for the parameters, the prior distributions being a key factor in the BVAR approach. Therefore, the advantages and disadvantages of VAR model and BVAR were listed in Table 1.

When developing the BVAR model, Litterman has made some assumptions on the unrestricted VAR model given by the following equations (Volkan and Gu, 2009):

$$Y_t = \mu + \Pi_1 y_{t-1} + \Pi_2 y_{t-2} + \cdots + \Pi_p y_{t-p} + \epsilon_t \tag{22}$$

$$y_t - y_{t-1} = c + \epsilon_t \tag{23}$$

As for example, writing the nth equation in a BVAR model:

$$\begin{aligned} Y_{i,t} &= c_i + \Phi_{i1}^1 y_{1,t-1} + \Phi_{i2}^1 y_{2,t-1} + \Phi_{in}^1 y_{n,t-1} \cdots \\ &+ \Phi_{i1}^2 y_{1,t-2} + \Phi_{i2}^2 y_{1,t-2} + \cdots + \Phi_{in}^2 y_{n,t-2} \\ &+ \cdots + \Phi_{n1}^p y_{1,t-p} + \Phi_{i2}^p y_{n,t-p} + \Phi_{in}^p y_{n,t-p} + \epsilon_{n,t}. \end{aligned} \tag{24}$$

The Φ_{ij}^s gives the coefficient relating y_{it} to $y_{j,t-s}$ Litterman (1980) assumed that $\Phi_{ii}^1 = 1$ and all the other $\Phi_{ij}^{(s)} = 0$. These 0 and 1 values characterize the mean of the prior distribution for the coefficients. Moreover, Litterman (1980) assumed that:

$$\Phi_{ii}^{(1)} \sim N\left(1,\gamma^2\right) \Phi_{ii}^{(s)} \sim N\left(1,\frac{\gamma^2}{s^2}\right) \Phi_{ij}^{(s)} \sim N\left(0,\left[S(i,j,l)\right]^2\right) \tag{25}$$

Although each equation $i = 1, 2,..., n$ of the VAR is estimated separately, the same value γ is used for each i. Smaller values of γ mean greater confidence in the prior information.

According to Litterman (1986), the prior for the variance is the only other prior to be set, the standard error on the coefficient estimate of lag l of variable j in equation i is denoted by a standard deviation of the form $S(i, j, l)$:

$$S(i,j,l) = \frac{\left[\gamma g(l) f(i,j)\right](\sigma_i)}{\sigma_i} \tag{26}$$

and:

$$f(i,j) = \begin{cases} 1 \ when \ i = j, \\ w_{ij} \ \ when \ i \neq j,. \end{cases}$$

where, $\frac{\sigma_i}{\sigma_j}$ is correction for the scale for the series i compared with j and $0 < \gamma < 1$. In Equation (26), the model requires choosing specific values for $g(l)$ (the lag decay) and γ, the tightness parameter and the standard deviation on the first own lag, will improve forecasting performance. Thus, the parameter $g(l)$ measures the tightness on lag l with respect to lag 1 and is assumed to have a harmonic shape with a decay factor of λ. (See Gupta and Sichei, 2006).

Litterman (1984a) a found that tight priors around zero on coefficients of other variables provide better forecast. Choon-Shan and Roy (2004) recommended a value of $\lambda = 0.7$ in concert with $\gamma = 0.9$. Kinal and Ratner (1986) used $\lambda = 0.40$ and $\gamma = 0.90$ (See Choon-Shan and Roy, 2004).

Table 1: Comparison of Bayesian VAR and VAR models

	Advantages	Disadvantages
Bayesian VAR model	Reducing root mean square imposing some prior restrictions on parameters percent error Easier and more accurate assessment of uncertainty.	Overcome problems with prior distributions
VAR model	Fairer assumptions about data interaction of different related variables in forecasting macroeconomic variable	Over-parameterization the loss of degrees of freedom which exponentially decrease for the number of lags included over-fitting phenomenon

VECM Model

VECMs are a theoretically-driven approach useful for estimating both short-term and long-term effects of one time series on another. Inherent in the distinction is the notion of equilibrium. Failure to establish co-integration often leads to spurious regression problems.

The *VECM*(*P*) form with the cointegration rank $r(\leq k)$ is written as:

$$\Delta LLMV_t = \delta + \pi LLMV_{t-1} + \Phi \Delta LLMV_{t-1} + \epsilon_t \quad (27)$$

where, ϵ_t is Gaussian random variable and are matrices of parameters estimated using OLS. The component π produces different linear combinations of levels of the time series X_t as such the matrix $LLMV_t$ contains information about the long run properties of the system describe by the model. For instance, if the rank of the matrix π is 0, then no series of the variables can be expressed as a linear combination of the remaining series. This indicates that there does not exists a long run relationship among the series of the VAR model as a test of co-integration a rank of 0 means integration is rejected. On the other hand, if the rank of the coefficient matrix π is 1, or greater than 1 then there exists 1 or more co-integrating vectors. This indicates a long run relationship or that the series exhibits significant evidence or behaving as a co-integrated system (See Kamal, 2014).

Since the Johansen approach requires a correctly specified VECM, it is necessary to ensure that the residuals in the model are "white noise". This involves setting the appropriate lag-length in the model and including (usually dummy) variables that only affect the short-run behavior of the model. It is pointed out that residual misspecification can arise as a consequence of omitting these important conditioning variables and increasing the lag-length is often not the solution (as it usually is, for example, when autocorrelation is present). The procedures for testing the properties of the residuals are discussed and illustrated through examples. We then consider the method of testing for "reduced rank", that is, testing how many co-integration vectors are present in the model. At this stage a major issue is confronted which presents considerable difficulty in applied work, namely that the reduced rank regression procedure provides information on how many unique co-integration vectors span the co-integration space, while any linear combination of the stationary vectors is itself also a stationary vector (See Richard, 1995).

Forecasting Accuracy

In this paper, the author focus on statistical methodology and forecast macroeconomic variable on time series datasets regarding real estate scenario. The Table 2 showed all the potential univariate forecasting models. However, the Log transformation reduced the Root Mean Square Error (RMSE) significantly, which is almost near to 0.1.

We mainly evaluate forecasting models based on the two performance measures of RMSE and Akaike Information Criterion (AIC) for univariate forecasting model. As was the case with the forecast in Table 4, land market value is projected to continue increase in the following years. It shows the stable increase in the future. This number will significantly rise to 7.3 in 2018.

Table 2: Comparison Table for univariate forecasting model

Model	ME	RMSE	MAE	MPE	MAPE	AIC
Regression	-1.103e-13	1127.403	918.1104	0.6624	21.1810	586.37193
Log Regression	-2.7e-17	0.1661	0.1489	-0.0371	1.7665	-12.84073
Regression with AR (2) errors	-5.2326	590.5238	424.728	-1.1753	7.9002	545.45
ETS (M,A,N)	-91.4374	617.2248	339.1251	-0.8840	408.4312	413.6146
ARIMA (1,1,0)	88.7736	713.7366	477.83	2.6121	7.973	532.83
Log ARIMA (2,0,1)	0.001	0.063	0.057	0.006	0.692	N/A

Table 3: Comparison Table for multivariate forecasting model

Model	VAR (1)	BVAR (2)	VECM (2)	AR(2)-GARCH(1,1)
RMSE	0.08125	0.00797	0.0059	0.139570
AICC	-860.1579	-4.5159	-12.8474	-39.04328
HQC 1		-4.49395	-12.8065	-39.031194
AIC -		4.52431	-12.8951	-39.04328
SBC 2		-4.43271	-12.6122	-32.99622
FPEC 3		0.010844	2.514E-6	
BIC 4	-805.7328			

The author measured forecasting performance by RMSE and AIC, as the authors did and we will also compare performance in terms of Mean absolute Error (MAE), AICC, Bayesian Information Criterion (BIC) and so on. The MAE criterion is most appropriate when the cost of a forecast error rises proportionally with respect to the absolute size of the error. With RMSE, the cost of the error rises as the square of the error and so large errors can be weighted far more than proportionally.

In addition, while RMSE and AIC are good relative measures, both depend on the scale of the forecast variable. Moreover, each could hypothetically be quite low and still contain systematic bias and do a poor job of forecasting average value changes. A given forecasting model may have a systematic positive or negative bias and do a poor job of tracking the actual mean of value changes and measures such as RMSE and MAE could well miss this defect. We will thus evaluate forecasts based on the four performance measures of RMSE, AIC and MAE.

In Table 3, we provide the forecasting accuracy for multivariate forecasting model. But some criterion are missing and not comparable. Eventually, we selected VECM (2) for forecasting land market value. Because the VECM (2) minimize the RMSE and AICC (the small-sample-size corrected version of Akaike information criterion).

Conclusion

Land market value, both directly and indirectly, is related to the housing market, commercial and residential buildings, construction industry, job-hunting market and home price. Therefore, the improved forecasting promise important benefits for any parties exposed to housing market (Miles, 2008).

The Table 4 shows the VECM (2) forecasting model for the general tendency of the land market value for about 10 years. The forecasting results shows the stable increase in the future. This number will increase to 9.12 in 2022.

The forecasting in real estate market is more important and necessary for the economy of American, because the tendency of Land Market Value would be helpful for government and investor to examine the problem in housing market, make the appropriate policy and regulate the housing market. On the other hand, forecasting techniques are widely used in the area of finance and housing market. Thus, a given forecasting model did a good job of tracking the actual value of land market changes. Most importantly, the advanced forecasting model can improve the accuracy and provide a better accurate guidance and more options for decision-maker.

Table 4: Forecasting VECM (2) Model for LLMV

Year	Forecasting	Error	95% L B	95% U B
2013	8.61155	0.07584	8.46290	8.76019
2014	8.66936	0.15104	8.37334	8.96539
2015	8.76250	0.22786	8.31590	9.20910
2016	8.86550	0.30301	8.27161	9.45939
2017	8.96023	0.37445	8.22633	9.69413
2018	9.03590	0.44069	8.17217	9.89963
2019	9.08805	0.50070	8.10670	10.06941
2020	9.11704	0.55394	8.03134	10.20275
2021	9.12638	0.60040	7.94961	10.30315
2022	9.12117	0.64054	7.86573	10.37661

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Ethics

This research paper follows the major requirement of ethics, stressing accuracy influence on advanced forecasting model. It also meet the required accurate definition of key concepts of ethical reflection such as principles and norms, values and virtues, rights and duties and the right perception of the implication of each one.

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Appendix[a]: 1982-2015 Land Market Value Datasets[b]

Year	LMV[c]	CPI	GDP[d]	IR	UR	CCI	PP	PMI	Year	LMV	CPI	GDP	IR	UR	CCI	PP	PMI
1982	1274.88	96.5	6.49	6.2	9.7	43.4	231.66	42.8	2000	4509.19	172.2	12.68	3.4	4	75.9	282.16	43.9
1983	1232.25	99.6	7	3.2	9.6	44.70	233.79	69.9	2001	5428.39	177.1	12.71	2.8	4.7	79.7	284.97	45.3
1984	1387.16	103.9	7.4	4.3	7.5	46.7	235.82	50.6	2002	6123.09	179.9	12.96	1.6	5.8	81.7	287.63	51.6
1985	1546.45	107.6	7.71	3.6	7.2	47.9	237.92	50.7	2003	7208.82	184.13	13.53	2.3	6	85.9	290.11	60.1
1986	1879.09	109.6	7.94	1.9	7	50.4	240.13	50.5	2004	8646.18	188.9	13.95	2.7	5.5	93.1	292.81	57.2
1987	2297.13	113.6	8.29	3.6	6.2	2.7	242.29	61	2005	10708.93	195.3	14.37	3.4	5.1	100	295.52	55.1
1988	2678.79	118.3	8.61	4.1	5.5	54.5	244.50	56	2006	12547.31	201.6	14.72	3.2	4.6	106	298.38	51.4
1989	3097.56	124.8	8.85	4.8	5.3	56.4	246.82	47.4	2007	12290.28	207.3	14.99	2.8	4.6	107	301.23	49
1990	3257.63	130.7	8.91	5.4	5.6	58	249.62	40.8	2008	10464.64	215.3	14.58	3.82	5.8	103.3	304.09	33.1
1991	3050.34	136.2	9.02	4.2	6.8	58.2	252.98	46.8	2009	7537.82	214.5	14.54	-0.32	9.3	98.10	306.77	55.3
1992	3089.8	140.3	9.41	3	7.5	58.9	256.51	54.2	2010	7173.83	218.1	14.94	1.64	9.6	96.4	309.35	57.5
1993	2948.23	114.5	9.65	3	6.9	61.8	259.92	55.6	2011	6184.28	224.9	15.19	3.14	8.9	97.4	311.72	53.1
1994	2995.76	148.2	10.05	2.6	6.1	64.6	263.13	56.1	2012	5543.56	229.6	15.43	2.08	8.1	98.4	314.11	50.4
1995	2945.05	152.4	10.28	2.8	5.6	67.3	266.28	46.2	2013	6777.04	233	15.92	1.46	7.4	104.8	316.5	56.5
1996	3033.87	156.9	10.74	3	5.4	68.6	269.39	55.2	2014	8152	237.2	16.29	1.61	6.2	111.8	318.86	55.1
1997	3120.62	160.5	11.21	2.3	4.9	70.6	272.65	54.5	2015	8737.11	242.1	16.3	0.1	5.5	100.37	320.99	53.5
1998	3437.02	163.11	11.77	1.6	4.5	72.5	275.85	46.8									
1999	3886.17	166.6	12.32	2.2	4.2	72.7	279.04	57.8									

[a]The data was based on the 34 years' national data on past and present real estate transaction from 1982 to 2015.
[b]http://www.statista.com/statistics/188105/annual-gdp-of-the-united-states-since-1990/
Source: U.S. Bureau of Labor Statistics https://en.wikipedia.org/wiki/Main-Page.
[c]The unit of land market value is million
[d]The unit of GDP is trillion.